University of Guelph
Department of Mathematics and Statistics

STAT*2040
Statistics I

Final Exam (White Version)
April 14 2011

Examiner: Jeremy Balka

This exam is two hours in duration

Name:

ID:

Signature:

Please read the instructions:

1. Fill out your name and ID number above.

2. When the examination starts, make sure your question paper is complete. You should have 39 multiple choice questions, along with formula sheets and statistical tables. The first question is just a bookkeeping question, and does not count for marks, but please fill it in to ensure your exam is properly graded.

3. Do all rough work on this paper.

4. You are allowed to bring in a calculator, and pens and pencils.

5. There is only **one** correct answer for each question. Fill in only one bubble for each question.

6. Fill out the computer answer sheet in pencil as you go. *There will be no extra time given at the end of the exam to fill in the sheet.*

7. The answers given in the exam are often rounded versions of the correct answer. Choose the closest value.

1. The colour of the first page of this examination booklet is:

   (a) White
   (b) Yellow

2. Which one of the following statements is *false*? (If A–D are all true, answer option E)

   (a) A statistic is a numerical characteristic of a sample, and a parameter is a numerical characteristic of a population.
   (b) The value of a statistic would vary in repeated sampling.
   (c) In most practical situations, the sampling distribution of a parameter will be approximately normal for large sample sizes.
   (d) $\hat{p}$ is a statistic, and the standard deviation of its sampling distribution depends on the parameter p.
   (e) None of the above.

3. Suppose we carry out a hypothesis test and find a p-value of .06. The evidence against H_0 is:

   (a) Significant at the 1% significance level.
   (b) Not significant at the 1% level, but significant at the 5% level.
   (c) Not significant at the 5% level, but significant at the 10% level.
   (d) Not significant at the 10% level.

4. A researcher uses a statistical computing package to carry out the calculations for a one-sample t test, and obtains the following output.

   t = XXXX, df = 38, p-value = 0.12

   alternative hypothesis: true mean is not equal to 10

   The t statistic has been blocked out, but I can tell you that the sample mean is equal to 3.2. The statistical computing package assumed a two-sided alternative hypothesis when calculating the p-value, but the researcher believes a one-sided alternative is more appropriate. What is the p-value of the one-sided alternative H_a: $\mu > 10$?

   (a) .06
   (b) .12
   (c) .24
   (d) .88
   (e) .94

5. Suppose we intend on drawing a simple random sample of size $n = 100$ from a large population. This population is normally distributed with a mean of 50 and a standard deviation of 400. Consider the following statements.

   The sampling distribution of the sample mean:

   I. Has a mean of 0.5.
   II. Has a standard deviation of 400.
   III. Is symmetric.

   Which of these statements are true?

   (a) Just II
   (b) Just III
   (c) Just II and III
   (d) All of them
   (e) None of them

6. Suppose we wish to use inference procedures based on the t distribution to perform a hypothesis test regarding a population mean. The population is strongly skewed to the right. Under which of the following situations would the use of the inference procedures based on the t distribution be the most reasonable?

   (a) $\bar{X} = 18.4, s = 1.1, n = 25$
   (b) $\bar{X} = 24.4, s = 8.1, n = 250$
   (c) $\bar{X} = 34.4, s = 18.1, n = 50$
   (d) $\bar{X} = 91.2,\ s = 3.1,\ n = 20$
   (e) $\bar{X} = 91.2,\ s = 6.1,\ n = 40$

7. Which one of the following statements is true?

   (a) In one-way ANOVA, the greater the F statistic the greater the p-value.
   (b) In a χ^2 test for count data, the greater the value of the test statistic, the greater the p-value.
   (c) The probability of a Type I error is always less than the probability of a Type II error.
   (d) The standard error of a statistic is equal to the margin of error of its 95% confidence interval.
   (e) None of the above.

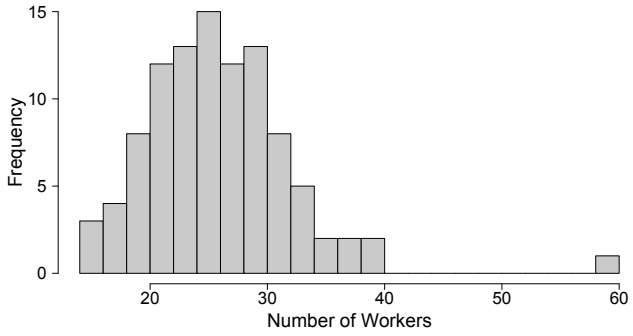
8. For two events A and B, it is known that $P(A) = 0.20$, $P(B) = 0.40$, and the probability that A or B or both occurs is .50. Which one of the following statements is *false*? (If A–D are all true, answer Option E).

   (a) $P(B|A) = .50$.
   (b) $P((A \cap B)^c) = .90$.
   (c) A and B are not mutually exclusive.
   (d) A and B are not independent.
   (e) None of the above.

9. Suppose the weight of adult green sea urchins (*strongylocentrotus dreobachiensis*) is approximately normally distributed with a mean of 48.0 grams and a standard deviation of 18.0 grams. The 30th percentile of weight of adult green sea urchins is closest to which one of the following?

   (a) 33
   (b) 36
   (c) 39
   (d) 42
   (e) 57

10. In an effort to make operations more efficient, a fast food chain is investigating the number of employees at each of its locations. A sample of 100 locations yielded the following plot.



   Which one of the following statements is true?

   (a) The interquartile range is greater than 20.
   (b) The standard deviation would be greater than 20.
   (c) Q_1 is greater than 20.
   (d) The class with the greatest frequency has a cumulative frequency that is greater than 30.
   (e) None of the above.

11. Suppose the random variable X has the following probability distribution.

x	2	4	6	8
$p(x)$	0.1	0.2	0.2	?

Find the missing probability, then calculate the mean of the random variable X. What is the mean of the random variable X?

(a) 2.2
(b) 4.4
(c) 5.0
(d) 5.5
(e) 6.2

12. In the inference procedures used in this course, we have sometimes used the t distribution, and sometimes the standard normal distribution. Consider the following four statements that relate to these two distributions.

I. In inference procedures for a single population mean μ, the t distribution should be used when the population variance is known.
II. The variance of the t distribution is greater than that of the standard normal distribution.
III. The t distribution tends toward the standard normal distribution as the degrees of freedom increase.
IV. The standard normal distribution is symmetric about 0, but the t distribution is not.

How many of these statements are true?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

13. In a certain area, a sample of 14 adult house sparrows was drawn. The average humerus (bone arm) length was found to be 1.60 cm, with a standard deviation of .045 cm. Assume that it is reasonable to think of these 14 sparrows as a random sample of adult sparrows from the area, and that the humerus lengths are approximately normally distributed. A 95% confidence interval for the population mean humerus length is closest to which one of the following?

    (a) $1.60 \pm .022$
    (b) $1.60 \pm .024$
    (c) $1.60 \pm .026$
    (d) $1.60 \pm .028$
    (e) $1.60 \pm .030$

14. Suppose that the 95% interval in the problem above was found to be (1.48, 1.72). This is not the correct interval but assume it is for the purposes of this question. Which one of the following is the most appropriate interpretation of this confidence interval?

    (a) In repeated sampling, 95% of the confidence intervals calculated in this manner will contain the mean humerus length of the 14 sparrows in the sample.
    (b) 95% of adult sparrows in the area have a humerus length between 1.48 and 1.72 centimetres.
    (c) In repeated sampling, 95% of sparrows in a random sample will have a humerus length between 1.48 and 1.72 centimetres.
    (d) In repeated sampling, 95% of the confidence intervals calculated in this manner will contain the true mean humerus length of adult sparrows in this area.
    (e) In repeated sampling, 95% of the sample means will fall between 1.48 and 1.72 centimetres.

15. A researcher wishes to investigate the mean of a population. The population standard deviation is known, but the mean is unknown. The researcher draws a random sample of 600 observations, and finds a sample mean of 2200. They run a test of the null hypothesis that the population mean is 1000 against the alternative it is greater than 1000, using a Z test. The researcher obtains a p-value of .08. Of the following options, which one best describes the meaning of that p-value?

    (a) If the null hypothesis is true, the probability of obtaining a sample mean at least as large as 2200 is 0.08.
    (b) If the null hypothesis is false, the probability of obtaining a sample mean at least as large as 2200 is 0.08.
    (c) The probability that the population mean is greater than 1000 is .08.
    (d) The probability that the null hypothesis is false is 0.08.
    (e) The probability that the null hypothesis is true is 0.08.

16. Suppose it is thought that the eggshell thickness for a certain species of bird in a certain area has a median of .40 mm. A random sample of 12 eggs from this type of bird are found to have shell thicknesses of:

.32, .32, .33, .33, .34, .36, .36, .36, .37, .37, .45, .55

Suppose we wish to use the sign test to test the null hypothesis that the population median shell thickness is .40, against a two-sided alternative hypothesis. The p-value of this test is closest to which one of the following?

(a) .01
(b) .02
(c) .03
(d) .04
(e) .05

17. A researcher uses a statistical computing package to carry out a one-sample t test, and obtains the following output.

t = 3.20, df = 6 p-value = XXXX

alternative hypothesis: true mean is not equal to 10

The p-value has been blocked out. One of the following 5 values is the appropriate p-value. Which one is it? (You have enough information to come up with the correct response)

(a) .0007
(b) .001
(c) .009
(d) .019
(e) .051

18. Suppose we wish to calculate a 45% confidence interval for the mean of a normally distributed population, in which the population standard deviation is known to be 25.0. If the sample size is $n = 100$, what is the appropriate margin of error of the interval?

(a) 1.2
(b) 1.5
(c) 2.2
(d) 3.7
(e) 4.5

N.B. The next two questions refer to the following information.

According to a very large survey in the U.S. in 2000, 14.1% of 18-25-year-olds were current users of marijuana or hashish. Even though this number is based on sample data, *assume here that* 14.1% *is a fixed, known parameter value for that time.* In a recent survey of 1000 randomly selected 18 to 25-year-olds, 184 of the 1000 surveyed currently use marijuana or hashish.

19. Suppose we wish to use the data from the recent sample to construct a 95% confidence interval for population proportion of 18 to 25-year-olds that currently use marijuana or hashish. What is the margin of error of the interval?

    (a) .020
    (b) .024
    (c) .028
    (d) .032
    (e) .036

20. Suppose we wish to test the null hypothesis that the population proportion is unchanged from the year 2000, against a two-sided alternative hypothesis. The value of the appropriate test statistic is closest to which one of the following?

    (a) 3.1
    (b) 3.3
    (c) 3.5
    (d) 3.7
    (e) 3.9

21. Which one of the following statements is true?

    (a) A p-value is the probability of a Type II error.
    (b) If in a hypothesis test the p-value is equal to exactly 1, the null hypothesis may still be false.
    (c) In one-way ANOVA, we assume that all of the groups have exactly the same sample standard deviation.
    (d) Both the t and χ^2 distributions tend toward the standard normal distribution as the degrees of freedom increase.
    (e) None of the above.

N.B. The next three questions refer to the following information.

In an experiment investigating the effect of calorie-restricted diets on the longevity of mice, mice were randomly assigned to five different diets (4 different calorie-restricted diets, and a standard diet, acting as a control group). Six mice were assigned to each of the diets (30 mice in total). The researchers felt the assumptions of one-way ANOVA are reasonable for this data. Consider the following partially-completed ANOVA table.

Source	DF	SS	MS	F
Treatments		12000		
Error				
Total		13400		

22. The value of the F test statistic is closest to which one of the following?

(a) 1
(b) 3
(c) 20
(d) 41
(e) 54

23. Which one of the following statements is false?

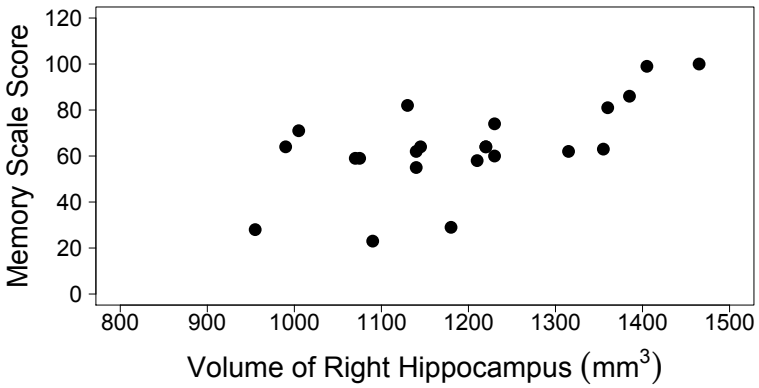
(a) There is significant evidence of a difference in mean longevity between the groups (at $\alpha = .05$).
(b) The pooled sample variance is equal to 56.
(c) The sample means were not all equal.
(d) This was a balanced experimental design.
(e) None of the above.

24. Refer again to the information above. Suppose we wish to calculate 95% confidence intervals for the differences in group means ($\mu_1 - \mu_2$, $\mu_1 - \mu_3$, etc.), using the LSD procedure. Which one of the following would be the appropriate margin of error of the intervals? (Choose the closest value).

(a) 8.2
(b) 8.5
(c) 8.9
(d) 9.7
(e) 10.2

N.B. The next 3 questions refer to the following information.

Bremner et al. (1995) investigated a possible relationship between memory and the volume of the right hippocampus (an area of the brain) in patients with post-traumatic stress disorder (PTSD). Twenty-two veterans of the Vietnam war had the volume of their right hippocampus measured by an MRI scanner. They were also assigned a score on their verbal memory according to the Wechsler Memory Scale. The following plot and output represent the results of a regression of Memory Scale score on right hippocampus volume.




```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -41.72277   29.37690  -1.420  0.17093
hipp_volume   0.08835    0.02440   3.621  0.00170

Residual standard error: 15.7 on 20 degrees of freedom
Multiple R-squared: 0.396,Adjusted R-squared: 0.3658
F-statistic: 13.11 on 1 and 20 DF,  p-value: 0.001704
```

25. Which one of the following is a 95% confidence interval for β_1?

    (a) $(.033, .142)$
    (b) $(.037, .139)$
    (c) $(.041, .136)$
    (d) $(.044, .133)$
    (e) $(.048, .129)$

26. The first observation in the data set had a right hippocampus volume of 955 mm^3 and a Memory Scale score of 28. The value of the residual associated with this value is closest to which one of the following?

    (a) -15
    (b) -10
    (c) -5
    (d) 0
    (e) 5

27. Consider the following statements:

I. There is strong evidence of a relationship between right hippocampus volume and score on the Wechsler Memory Scale.
II. More than 50% of the variation in Memory Scale score can be explained by the linear relationship with right hippocampus volume.
III. The null hypothesis that corresponds to the p-value of .00170 in the output is H_0: $\hat{\beta}_1 = 0$.
IV. The correlation coefficient (r) is positive.

How many of these four statements are true?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

28. Consider the following statements about simple linear regression and correlation. Which statement is true?

(a) The parameter estimates are chosen such that $\sum (X_i - (\beta_0 + \beta_1))^2$ is a minimum.
(b) The sum of the residuals in a simple linear regression is always equal to n.
(c) If the residuals are all equal, then $s^2 = 0$.
(d) If $\beta_0 = 0$, then there is no linear relationship between X and Y.
(e) None of the above.

29. Which one of the following statements about one-way ANOVA is true?

(a) If the null hypothesis is false, then the F statistic will always be large.
(b) If the sample means of all the groups are exactly equal, then $F = 1$.
(c) An F statistic that is very close to 0 implies there is very strong evidence against the null hypothesis.
(d) The alternative hypothesis is that the population variances are not all equal.
(e) None of the above.

N.B. The next 2 questions refer to the following information.

A study investigated two subspecies (migratory, nonmigratory) of the dark-eyed junco (*Junco Hyemalis*), a type of American sparrow. One variable that was measured on each subspecies was wing length. A random sample of 10 migratory dark-eyed juncos had a mean wing length of 82.7 mm, with a standard deviation of 1.5 mm. An independent random sample of 20 nonmigratory dark-eyed juncos had a mean wing length of 85.1 mm, with a standard deviation of 1.7 mm.
The researchers decided to use a pooled variance t test to test the null hypothesis that the two subspecies have the same population mean wing length. The results of the t test are given in the following output.

```
 Two Sample t-test

data:  migratory and nonmigratory
t = -3.8699, df = XXX, p-value = XXX
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.622834 -1.114994
sample estimates:
mean of x mean of y
 82.74124  85.11016
```

30. The degrees of freedom and p-value have been XXXed out. Of the following options, which one is the best interpretation at a 5% significance level?

    (a) The mean wing length of the migratory and nonmigratory subspecies are significantly equal.
    (b) There is no significant difference between the mean wing length of the migratory and nonmigratory subspecies.
    (c) There is a significant difference in the mean wing length between the two subspecies.
    (d) There is an extremely important difference in the mean wing length between the two subspecies.
    (e) There is strong evidence that the sample means are equal.

31. Consider again the information in the previous question. Note that the pooled-variance t procedure was used in the analysis. Which one of the following statements is *true*?

    (a) The procedure used to obtain the output is a *distribution-free* procedure (the procedure does not require a normality assumption).
    (b) An assumption of the procedure is that the sample standard deviations are exactly equal.
    (c) Since the sample standard deviations are not equal, and the sample sizes are not equal, the pooled-variance t procedure is a very poor method of analysis.
    (d) The degrees of freedom for the procedure used in the previous question are 29.
    (e) Had we carried out a one-way ANOVA on the above data at the same level of significance, our conclusions would have been the same.

32. A researcher investigated a possible relationship between the gender and ethnicity of people with AIDS. A sample of 600 people living with AIDS in the U.S. were classified according to their gender and race/ethnicity, with the following results:

	Male	Female
White	143	29
Black	186	100
Hispanic	86	23
Other	26	8

(This question does not require a lot of calculations)
If we test the null hypothesis that gender and race/ethnicity are independent for AIDS patients in the U.S., we find that the value of the appropriate χ^2 test statistic is 20.4. Which one of the following is the best conclusion at $\alpha = .01$?

(a) There is significant evidence that gender and race/ethnicity are independent.
(b) There is significant evidence that knowing an AIDS victim's gender gives no information about their race/ethnicity.
(c) There is not significant evidence gender and race/ethnicity are not independent.
(d) There is significant evidence that gender and race/ethnicity are not independent.
(e) None of the above.

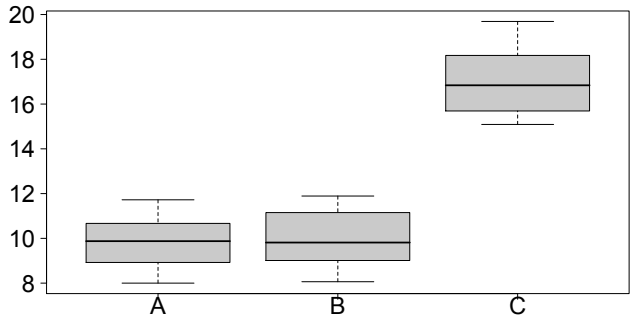
33. This question is very similar to a question from Assignment #9.
Suppose that a type of bird always lays exactly 3 eggs per year. This is of course very unlikely, but assume that it is true for the purposes of this question. A scientist claims that the number of eggs that hatch per nest should follow a binomial distribution. You wish to investigate this claim, and follow a sample of 500 nests until the eggs hatch. The results for the 500 nests are summarized in the following table.

# that hatch	0	1	2	3
Frequency	78	71	164	187

Suppose we wish to use the appropriate χ^2 test to test the scientist's claim that the number of eggs that hatch per nest follows a binomial distribution. For this question you only have to calculate a single expected count (but calculate it in the same way as if you had to carry out the complete test). The appropriate expected count of nests in which 3 eggs hatch is closest to which one of the following?

(a) 130
(b) 131
(c) 132
(d) 133
(e) 134

34. Consider the following 3 boxplots, representing 3 separate and independent samples of size 20.



Consider the following 4 statements:

I. If we performed a one-way ANOVA on these 3 samples, the p-value would be small.
II. If we carried out a t-test of H_0: $\mu_A = \mu_B$ against a two-sided alternative, the p-value would be small.
III. The use of the pooled-variance t procedures for the test of H_0: $\mu_A = \mu_C$ would be a bad idea, as the assumptions are clearly violated.
IV. The standard deviation of Sample C is greater than the mean of Sample C.

How many of these statements are true?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

35. A researcher wants to test whether the mean of a certain normally distributed population is 50, against the alternative that it is greater than 50. They are planning on drawing a random sample ($n = 100$). They know very little about statistics, and decide to simply reject the null hypothesis if the value of the sample mean is greater than 55. Suppose it is known that the population variance is 1600. What is the power of the test if the true population mean is 65? (Choose the closest value).

(a) .01
(b) .923
(c) .962
(d) .994
(e) .999

36. A researcher draws a random sample of 75 observations from a normally distributed population. The mean of the population is unknown, but the standard deviation of the population is known to be 6. The researcher wants to test the null hypothesis that the mean of the population is 12, against the alternative that it is different from 12. The researcher intends to use the test statistic:

$$\frac{\bar{X} - 12}{6/\sqrt{75}}$$

What is the sampling distribution of this test statistic if the null hypothesis is true?

(a) A standard normal distribution.
(b) A normal distribution with $\mu = 12$, and $\sigma = 6$.
(c) A t distribution with 75 degrees of freedom.
(d) A t distribution with 74 degrees of freedom.
(e) None of the above.

37. In the notation of our course, how many of the following letters or symbols represent *parameters*?
μ, p, s, $\hat{\beta}_0$, $\hat{\beta}_1$, $\bar{X}$

(a) 2
(b) 3
(c) 4
(d) 5
(e) 6

38. Forty patients with high blood pressure volunteer for a study. The participants are given an injection of a placebo (a saline solution with no pharmacological effect), and an hour later their drop in blood pressure is measured. Two days later the same 40 patients are given an injection of an experimental new drug, and an hour later the drop in blood pressure is recorded. Researchers want to investigate the effect of the drug on blood pressure. They wish to compare the drop in blood pressure after the injection of the drug to the drop in blood pressure after injection of the placebo. Which one of the following is the most appropriate test? (Assume normality where necessary).

(a) A χ^2 test with 1 degree of freedom.
(b) A Z test.
(c) A t test with 79 degrees of freedom.
(d) A t test with 78 degrees of freedom.
(e) A t test with 39 degrees of freedom.

39. This question involves estimation of a parameter θ. It is not a parameter that we have discussed in class, but you should be able to answer this question if you fully understood the methods discussed in our course.

A researcher wants to calculate a 95% confidence interval for a parameter θ. The statistic $\hat{\theta}$ is a point estimator of θ, and is more complicated than any we have discussed in class. The researcher draws a random sample ($n = 300$), and finds that $\hat{\theta} = 2.00$ for her sample. Suppose it is known that for samples of this size, the sampling distribution of $\hat{\theta}$ is normal, with a mean of θ, and a standard deviation of 3.00. Using the knowledge that you have learned in class, you should be able to calculate a 95% confidence interval for θ. What is the appropriate interval?

(a) $(-3.88, 7.88)$
(b) $(-1.39, 5.39)$
(c) $(-1.14, 5.14)$
(d) $(1.16, 2.34)$
(e) $(1.36, 2.14)$