1)

a) $f(x) = \frac{-x^3}{4} - 13x^2 + 13x - 9$

Type of Polynomial = Cubic (∵ highest degree is '3')

Sign of leading coefficient = negative (−ve) [∵ $-\frac{1}{4}$ is leading term coefficient]

End behaviours = as $x \to \infty$, $\frac{-x^3}{4} \to -\infty$

as $x \to -\infty$, $\frac{-x^3}{4} \to \infty$

∴ f(x) is down to right and up to the left

Domain is $x \in \mathbb{R}$ (∵ for, Every value of 'x', there is value for 'f(x)')

b) $f(x) = 6x^4 + 7x^2 - 8x + 9$

Type of Polynomial = Biquadratic (or) quartic (∵ highest degree is '4')

Sign of leading coefficient = Positive (+ve) [∵ +6 is leading coefficient of $6x^4$]

End behaviours = as $x \to \infty$, $6x^4 \to \infty$

as $x \to -\infty$, $6x^4 \to \infty$

∴ f(x) is Up to the right and Up to the left

Domain is $x \in \mathbb{R}$ (∵ Every value of 'x', has value of 'f(x)')

$f(x) = -(x-4)(x+1)^2(x-5)$

x-intercept $\to$ $y=0$ $\Rightarrow$ $-(x-4)(x+1)^2(x-5) = 0$

$x-4=0, \quad (x+1)^2 = 0, \quad x-5=0$

$x=4, \quad x=-1, \quad x=5$

$\therefore \boxed{(4,0)\ (-1,0)\ (5,0)}$

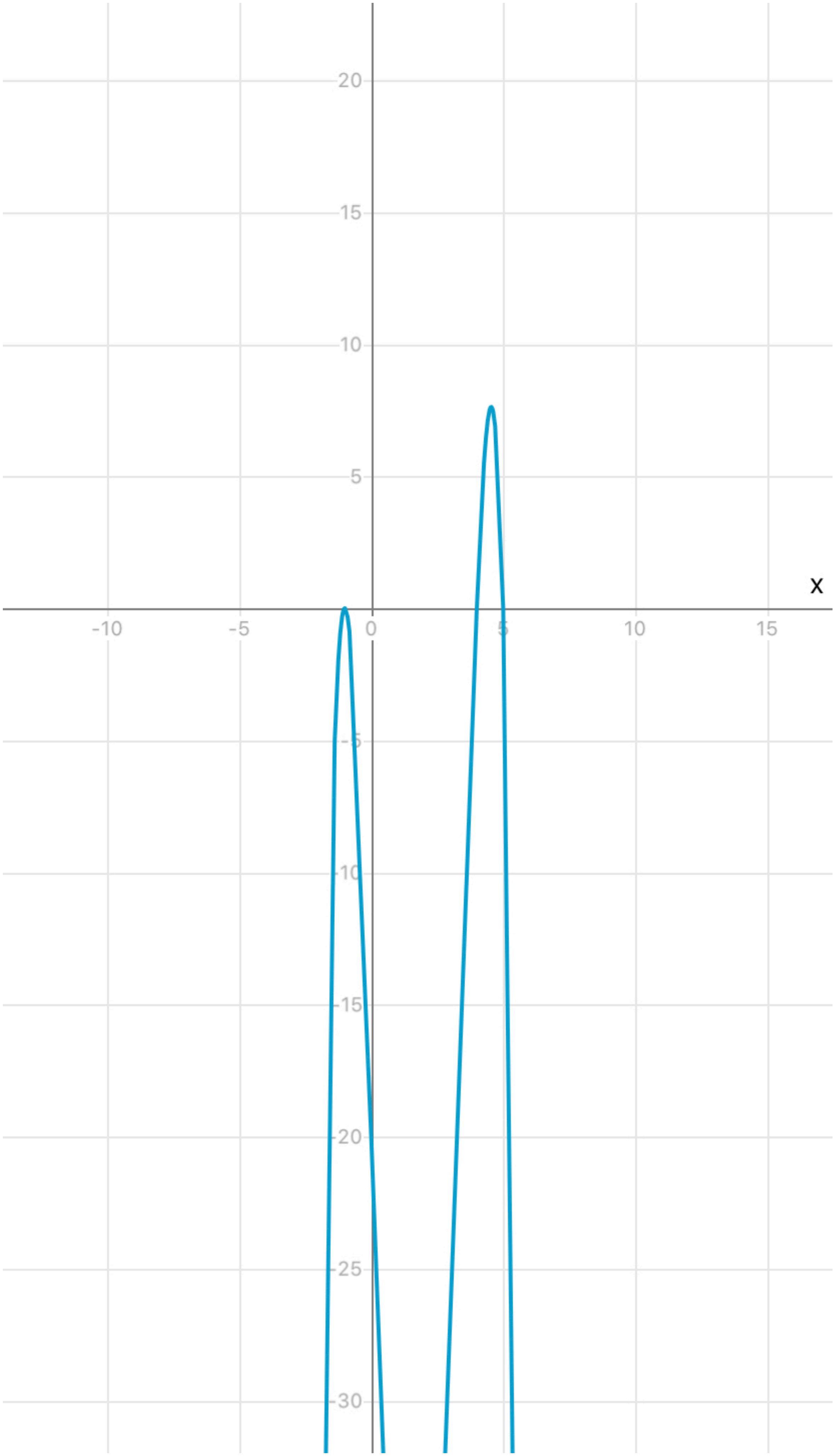
Y intercept $\to$ $x=0$ $\therefore$ $y = -(0-4)(0+1)^2(0-5) = -20$ $\therefore \boxed{(0,-20)}$

When $x \to \infty$

$$\lim_{x\to\infty} f(x) \Rightarrow \lim_{x\to\infty}\left[-(x-4)(x+1)^2(x-5)\right] = (-1)\lim_{x\to\infty}(x-4)(x+1)^2(x-5)$$

$$= (-1)(\infty) = -\infty$$

$$\therefore \lim_{x\to\infty} f(x) = \boxed{-\infty}$$



20
15
10
5
-5
-10
-15
-20
-25
-30
x
-10
-5
0
5
10
15

Solution: 3.

The equation is :

$$18x^4 + 87x^3 + 3x^2 - 108x = 0$$

Taking x common from each term we get

$$x\left(18x^3 + 87x^2 + 3x - 108\right) = 0$$

$$\therefore x = 0 \text{ or } 18x^3 + 87x^2 + 3x - 108 = 0$$

Consider the equation $18x^3 + 87x^2 + 3x - 108 = 0$.

We will use synthetic division to find its roots and we start with 1.

1	18	87	3	−108
		18	105	108
	18	105	108	0

Therefore, we get

Quotient in coefficient form $(18, 105, 108)$

In index form:

$= 18x^2 + 105x + 108$

$= 3\left(6x^2 + 35x + 36\right)$

$= 3\left(6x^2 + 27x + 8x + 36\right)$

$= 3\left[3x\left(2x + 9\right) + 4\left(2x + 9\right)\right]$

$= 3\left(3x + 4\right)\left(2x + 9\right)$

$\therefore 18x^3 + 87x^2 + 3x - 108 = 3\left(x - 1\right)\left(3x + 4\right)\left(2x + 9\right) + 0$

$\therefore 18x^3 + 87x^2 + 3x - 108 = 0$

$\Rightarrow 3\left(x - 1\right)\left(3x + 4\right)\left(2x + 9\right) = 0$

$\Rightarrow x = 1, -\frac{4}{3}, -\frac{9}{2}$

$\therefore$ The solution for the equation $18x^4 + 87x^3 + 3x^2 - 108x = 0$ is

$x = 0, 1, -\frac{4}{3}, -\frac{9}{2}$

(7)

a) The parent function: $g(x) = x^3$

Now, by horizontal translation, move to left by 5 units;

so, $g_1(x) = (x-5)^3$

Now, by horizontal dilation; by a factor of $\frac{1}{2}$

so, $g_2(x) = \left[\frac{1}{2}(x-5)\right]^3$

Now, by vertical dilation, by a factor of 7;

so, $g_3(x) = 7\left[\frac{1}{2}(x-5)\right]^3$.

Now, by vertical translation, move to down by 45 units.

so, $g_4(x) = f(x) = 7\left[\frac{1}{2}(x-5)\right]^3 - 45$

b) as $f(x) = 7\,g\left(\frac{1}{2}(x-5)\right) - 45$; $g(x) = \frac{1}{2}(x-5)$

[g(x) is the parent function]

as $(-9, -2446)$ is on $f(x)$.

So, $f(-9) = 7\,g\left(\frac{1}{2}(-9-5)\right) - 45$

$\Rightarrow -2446 = 7\,g(-7) - 45$

$\Rightarrow 7\,g(-7) = -2446 + 45 = -2401$

$\Rightarrow g(-7) = \frac{-2401}{7} = -343$

Thus, the original point is $\boxed{(-7, -343)}$

Please Rate !! :)

Sol. Using Remainder theorem

$$f(x) = mx^4 + nx^3 + 68x^2 - x - 6$$

divided by $(2x+1)$

$$2x+1 = 0$$

$$x = -\frac{1}{2}$$

Remainder $= f(-\frac{1}{2})$

$$0 = m\left(-\frac{1}{2}\right)^4 + n\left(-\frac{1}{2}\right)^3 + 68\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 6$$

{ Remainder $= 0$ given in question }

$$0 = \frac{m}{16} - \frac{n}{8} + \frac{68}{4} + \frac{1}{2} - 6$$

$$0 = \frac{m-2n}{16} + \frac{68+2-24}{4}$$

$$\frac{m-2n}{16} = \frac{24-70}{4}$$

$$m - 2n = 16\left(-\frac{46}{4}\right)$$

$$m - 2n = -184 \quad \text{------ (1)}$$

$f(x) = mx^4 + nx^3 + 68x^2 - x - 6$

divided by $(4x-1)$

$$4x - 1 = 0$$

$$x = \frac{1}{4}$$

Remainder $= f\left(\frac{1}{4}\right)$

$$0 = m\left(\frac{1}{4}\right)^4 + n\left(\frac{1}{4}\right)^3 + 68\left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right) - 6$$

$$0 = \frac{m}{256} + \frac{n}{64} + \frac{68}{16} - \frac{1}{4} - \frac{6}{1}$$

$$0 = \frac{m + 4n}{256} + \frac{68 - 4 - 96}{16}$$

$$\frac{m + 4n}{256} = \frac{100 - 68}{16}$$

$$\frac{m + 4n}{256} = \frac{32}{16}$$

$$m + 4n = 512 \quad \text{——— } (2)$$

from eq 1 & 2

$$\cancel{m} - 2n = -184$$

$$\underset{-}{\cancel{m}} \underset{-}{+} 4n = \underset{-}{512}$$

$$-6n = -696$$

$$\boxed{n = 116}$$

Value of n put in eq 2

$$m + 4(116) = 512$$

$$m + 464 = 512$$

$$\boxed{m = 48}$$

6 (a) Given function $f(x) = 55x^4 - x^2 - 78$.

Now $f(-x) = 55(-x)^4 - (-x)^2 - 78$.

$= 55x^4 - x^2 - 78$

$= f(x)$

$\therefore f(-x) = f(x)$

Hence $f(x)$ is an even function.

(b) Given that $f(x) = 37x^3 + 93x$

Now $f(-x) = 37(-x)^3 + 93(-x)$

$= -37x^3 - 93x$

$= -(37x^3 + 93x)$

$\therefore f(-x) = -f(x)$, $f(x)$ is an odd function