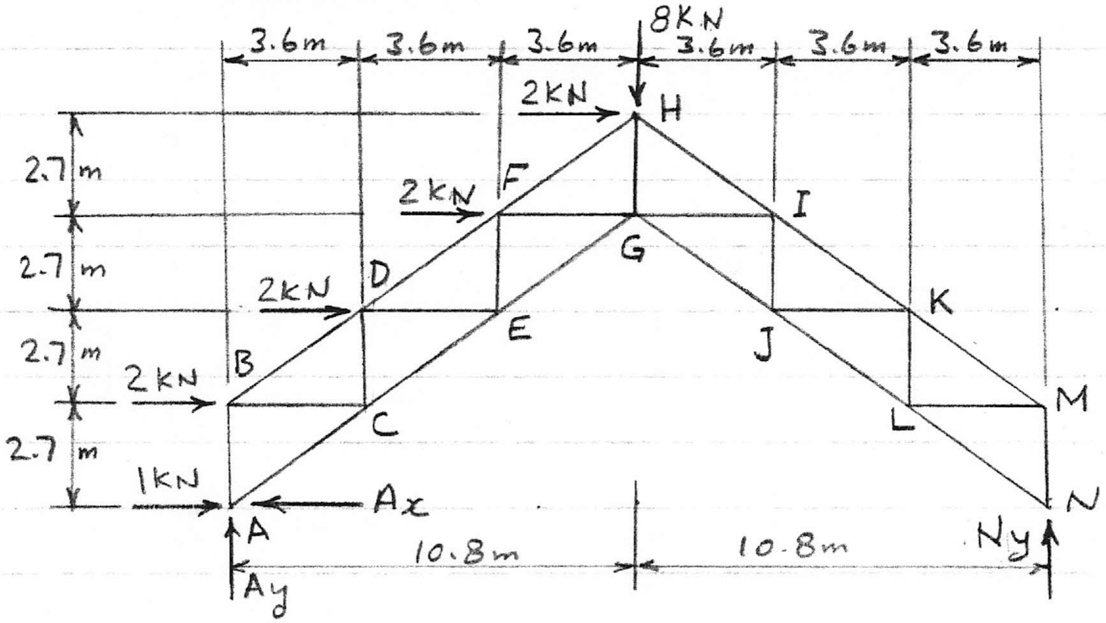
Dec. 9, 2009

# GNG 1105
# FINAL EXAMINATION
# SOLUTIONS

1.

a)

2




FBD – Entire truss

$+\circlearrowleft \Sigma M_A = 0,$

2 $N_y \times 21.6 - 2KN \times 2.7 - 2KN \times 5.4 - 2KN \times 8.1 - 2KN \times 10.8$

$- 8KN \times 10.8 = 0$

$21.6 N_y - 5.4 - 10.8 - 16.2 - 21.6 - 86.4 = 0$

$21.6 N_y = 140.4$

$\therefore N_y = \frac{140.4}{21.6} = \underline{\underline{6.5 KN \uparrow}}$ ANS.

1 $+\uparrow \Sigma F_y = 0,$

$A_y + N_y - 8KN = 0$

$A_y + 6.5KN - 8KN = 0, \quad \therefore A_y = \underline{\underline{1.5KN \uparrow}}$ ANS.

$\xrightarrow{+} \Sigma F_x = 0$

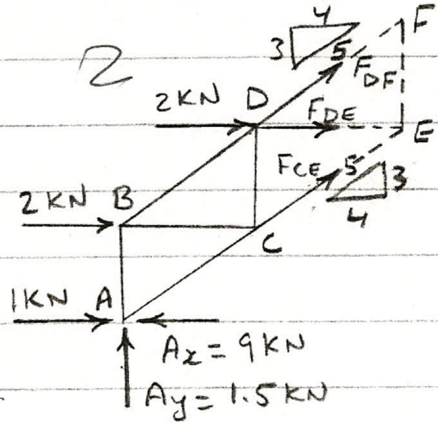
$- A_x + 1KN + 2KN + 2KN + 2KN + 2KN = 0$

$\therefore A_x = \underline{\underline{9KN}} \leftarrow$ ANS.

1 – (Cont'd)

b) FBD – Left side

2




$\overset{+}{\curvearrowleft} \sum M_D = 0$

$-1.5KN \times 3.6m - 9KN \times 5.4m$

$+ 1.0KN \times 5.4m + 2KN \times 2.7m$

2 $\quad + F_{CE} \times \frac{4}{5} \times 2.7m = 0$

$-5.4 - 48.6 + 5.4 + 5.4 + 2F_{CE} = 0$

$2F_{CE} = 43.2$

$\therefore F_{CE} = \frac{43.2}{2} = \underline{\underline{21.6KN}}\ (T)$ ANS.

$\overset{+}{\curvearrowleft} \sum M_E = 0$

1 $\quad -1.5KN \times 7.2m - 9KN \times 5.4m + 1KN \times 5.4m + 2KN \times 2.7m$

$- F_{DF} \times \frac{3}{5} \times 3.6m = 0$

$-10.8 - 48.6 + 5.4 + 5.4 - 2.16 F_{DF} = 0$

$2.16 F_{DF} = -48.6$

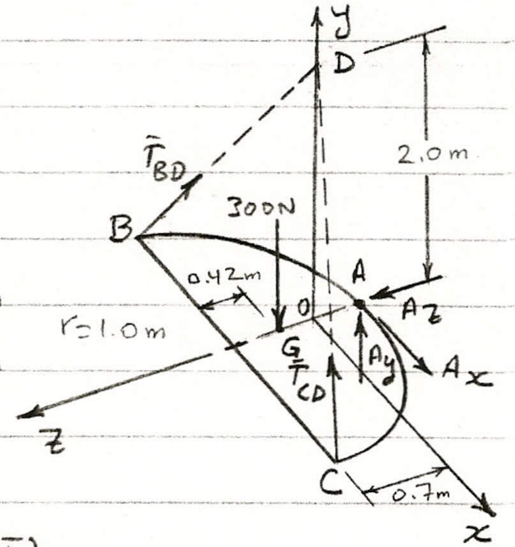
$\therefore F_{DF} = -\frac{48.6}{2.16} = -22.5KN = \underline{\underline{22.5KN}}\ (C)$ ANS.

$\overset{+}{\rightarrow} \sum F_x = 0$

$1KN - 9KN + 2KN + 2KN + 21.6 \times \frac{4}{5} - 22.5 \times \frac{4}{5} + F_{DE} = 0$

$1 - 9 + 2 + 2 + 17.28 - 18 + F_{DE} = 0$

$\therefore F_{DE} = \underline{\underline{4.72\ KN}}\ (T)$ ANS.




2.

a) FBD - See diagram

b)

$\bar{BD} = +1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}$ ; $BD = 2.34m$

$\bar{CD} = -1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}$ ; $CD = 2.34m$

$\bar{W} = -300N\,\bar{j}$

$\bar{T}_{BD} = T_{BD}\,\bar{\lambda}_{BD} = T_{BD}\,\dfrac{\bar{BD}}{BD}$

$= \dfrac{T_{BD}}{2.34}(1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k})$

$\bar{T}_{CD} = T_{CD}\,\bar{\lambda}_{CD} = T_{CD}\,\dfrac{\bar{CD}}{CD}$

$= \dfrac{T_{CD}}{2.34}(-1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}$

$\bar{w} = -300N\,\bar{j}$

c) $\Sigma\bar{M}_A = 0$

$\Sigma\bar{M}_A = \bar{r}_{B/A}\,\bar{T}_{BD} + \bar{r}_{C/A}\,\bar{T}_{CD} - \bar{r}_{G/A}\,(300N)\,\bar{j} = 0$

where $\bar{r}_{B/A} = -1.0m\,\bar{i} + 1.0m\,\bar{k}$

$\bar{r}_{C/A} = +1.0m\,\bar{i} + 1.0m\,\bar{k}$

$\bar{r}_{G/A} = +(1.0 - 042)\,\bar{k} = 0.58m\,\bar{k}$

$\therefore \Sigma\bar{M}_A = (-1.0\bar{i} + 1.0\bar{k}) \times \dfrac{T_{BD}}{2.34}(1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k})$

$+ (1.0\bar{i} + 1.0\bar{k}) \times \dfrac{T_{CD}}{2.34}(-1.0\bar{i} + 2.0\bar{j} - 0.7\bar{k}) - 0.58\bar{k} \times 300\bar{j} = 0$

$\Sigma\bar{M}_A = -2 \times \dfrac{T_{BD}}{2.34} - 0.7 \times \dfrac{T_{BD}}{2.34}\bar{j} + 1 \times \dfrac{T_{BD}}{2.34}\bar{j} - 2 \times \dfrac{T_{BD}}{2.34}\bar{i}$

$+ 2 \times \dfrac{T_{CD}}{2.34} + 0.7 \times \dfrac{T_{CD}}{2.34}\bar{j} - 1 \times \dfrac{T_{CD}}{2.34}\bar{j} - 2 \times \dfrac{T_{CD}}{2.34}\bar{i} + 174\bar{i} = 0$

i.e. $\Sigma M_A = -0.85 T_{BD}\bar{k} - 0.3 T_{BD}\bar{j} + 0.43 T_{BD}\bar{j} - 0.85 T_{BD}\bar{i}$

$+ 0.85 T_{CD}\bar{k} + 0.3 T_{CD}\bar{j} - 0.43 T_{CD}\bar{j} - 0.85 T_{CD}\bar{i} + 174\bar{i} = 0$

Equate Coeff. of $\bar{i}$, $\bar{j}$ & $\bar{k}$ to zero

$(\bar{i})$: $-0.85\,T_{BD} - 0.85\,T_{CD} + 174 = 0$ ; Because of Symmetry $T_{BD} = T_{CD}$

$\therefore 1.70\,T_{BD} = 174$

Hence, $T_{BD} = T_{CD} = \dfrac{174}{1.70} = \underline{\underline{102.35N}}$ ANS.

2 -

Another Method

$$\Sigma M_A = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 0 & +1 \\ +1 & +2 & -0.7 \end{vmatrix} \times \frac{T_{BD}}{2.34} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ +1 & 0 & +1 \\ -1 & +2 & -0.7 \end{vmatrix} \times \frac{T_{CD}}{2.34} - 0.58\bar{k} \times 300\bar{j} = 0$$

Coeff. of $(\bar{i})$: $\dfrac{-2T_{BD}}{2.34} - \dfrac{2T_{CD}}{2.34} + 174 = 0$

$$-0.85T_{BD} - 0.85T_{CD} + 174 = 0$$

$T_{BD} = T_{CD}$; $\therefore$ $1.70\,T_{BD} = 174$

Hence, $T_{BD} = T_{CD} = \dfrac{174}{1.70} = 102.35\,N$ ANS.

Coeff. of $(\bar{j})$: $-\dfrac{0.7T_{BD}}{2.34} + \dfrac{1T_{BD}}{2.34} + 0.7\dfrac{T_{CD}}{2.34} - \dfrac{1T_{CD}}{2.34} = 0$

$\dfrac{0.3T_{BD}}{2.34} - \dfrac{0.3T_{CD}}{2.34} = 0$; $\therefore$ $T_{BD} = T_{CD}$ ✓ check.

Coeff. of $(\bar{k})$: $-\dfrac{2T_{BD}}{2.34} + \dfrac{2T_{CD}}{2.34} = 0$, $\therefore$ $T_{BD} = T_{CD}$ ✓ check.

2. (Cont'd):

$(\bar{j})$: $-0.3T_{BD} + 0.43T_{BD} + 0.3T_{CD} - 0.43T_{CD} = 0$

$0.13T_{BD} - 0.13T_{CD} = 0$

$\therefore T_{BD} = T_{CD}$ ✓ Check.

$(\bar{k})$: $-0.85T_{BD} + 0.85T_{CD} = 0$

$\therefore T_{BD} = T_{CD}$ ✓ Check.

Components of reaction at A:

$\Sigma \bar{F}_x = 0$

$(\bar{i})$: $A_x + \frac{1.0}{2.34} \times T_{BD} - \frac{1.0}{2.34} \times T_{CD} = 0$

$A_x + \frac{102.35}{2.34} - \frac{102.35}{2.34} = 0$; $\therefore$ $\underline{\underline{A_x = 0}}$ ANS.

$(\bar{j})$: $A_y + \frac{2.0}{2.34} \times T_{BD} + \frac{2.0}{2.34} \times T_{CD} - 300\,N = 0$

$A_y + \frac{2.0}{2.34} \times 102.35 + \frac{2.0}{2.34} \times 102.35 - 300 = 0$

$A_y + 87.48 + 87.48 - 300 = 0$; $\therefore$ $\underline{\underline{A_y = +125.04\,N}}$ ANS.

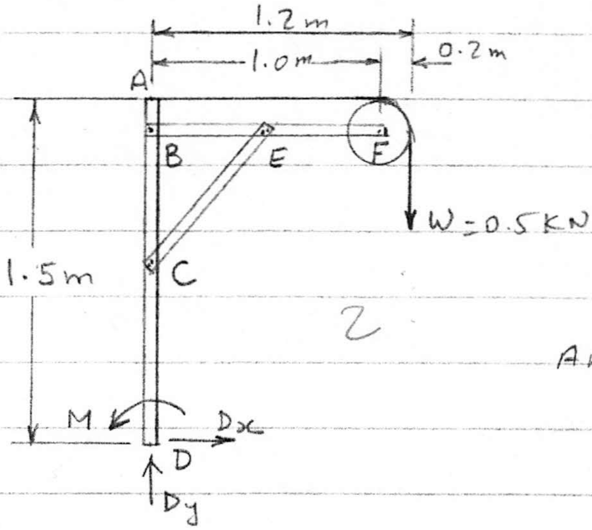
$(\bar{k})$: $A_z - \frac{0.7}{2.34} \times T_{BD} - \frac{0.7}{2.34} \times T_{CD} = 0$

$A_z - \frac{0.7}{2.34} \times 102.35 - \frac{0.7}{2.34} \times 102.35 = 0$

$A_z - 30.62 - 30.62 = 0$; $\therefore$ $\underline{\underline{A_z = +61.24\,N}}$ ANS.

3.

a) FBD - Entire frame




$\circlearrowleft + \Sigma M_D = 0$

$M - 0.5\,KN \times 1.2\,m = 0$ 2

$\therefore M = 0.6\,KN.m \circlearrowleft$ ANS.

2

$\xrightarrow{+} \Sigma F_x = 0$

$D_x = 0$ 1

$+\uparrow \Sigma F_y = 0$

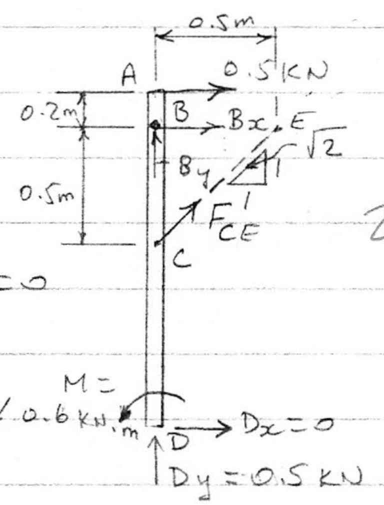
$D_y - 0.5\,KN = 0, \;\; \therefore D_y = 0.5\,KN \uparrow$ 1 ANS.

b) FBD - member ABCD

CE is a 2-force member.

$\circlearrowleft + \Sigma M_B = 0$

$-0.5\,KN \times 0.2\,m + F_{CE} \times \frac{1}{\sqrt{2}} \times 0.5\,m + 0.6\,KN.m = 0$




2

$-0.1\,KN.m + 0.35\,F_{CE} + 0.6\,KN.m = 0$

2 $0.35\,F_{CE} = -0.5$

$\therefore F_{CE} = -\frac{0.5}{0.35} = \underline{\underline{-1.43\,KN}} \swarrow$ ANS.

Since CE is a 2-force member, $\therefore F_{EC} = \underline{\underline{1.43\,KN}} \nearrow$ ANS.

$\xrightarrow{+} \Sigma F_x = 0$

$B_x + 0.5\,KN - 1.43 \times \frac{1}{\sqrt{2}} = 0$

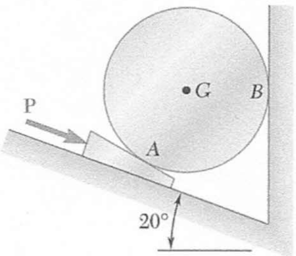
$B_x + 0.5 - 1.01 = 0; \qquad \therefore B_x = \underline{\underline{0.51\,KN}} \rightarrow$ ANS.

1

$+\uparrow \Sigma F_y = 0$

$B_y + 0.5\,KN - 1.43 \times \frac{1}{\sqrt{2}} = 0$

$B_y + 0.5 - 1.01 = 0; \qquad \therefore B_y = \underline{0.51\,KN} \uparrow$ 1



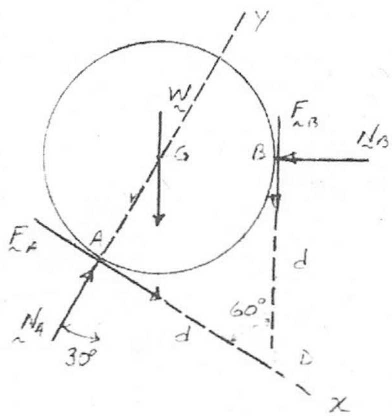

## PROBLEM 8.60

A 10° wedge is forced under an 80-kg cylinder as shown. Knowing that the coefficient of static friction at all surfaces is 0.25, determine the force **P** for which motion of the wedge is impending.

## SOLUTION

**FBD Cylinder:**

2




note $d = \dfrac{r}{\tan 30} = \sqrt{3}r$

$$W = (80\text{ kg})(9.81\text{ m/s}^2) = 784.8\text{ N}$$

$$\circlearrowleft \Sigma M_G = 0: \quad F_A - F_B = 0, \quad F_A = F_B \qquad (1)$$

$$\circlearrowleft \Sigma M_D = 0: \quad dN_B - dN_A + rW = 0, \quad N_A = N_B + \frac{W}{\sqrt{3}} \qquad 2 \qquad (2)$$

so $\quad N_A > N_B, \quad F_{A\max} > F_{B\max}$

$\therefore$ slip impends first at $B$. $F_B = \mu_s N_B = 0.25 N_B$

$$\circlearrowleft \Sigma M_A = 0: \ (r\cos 30°)N_B - (r\sin 30°)W - r(1+\sin 30°)(0.25 N_B) = 0$$

$$N_B = 1.01828W = 799.15\text{ N} \qquad 2$$

$$F_B = 0.25 N_B = 199.786\text{ N}$$

From (2) above, $N_A = 799.15\text{ N} + \dfrac{784.8\text{ N}}{\sqrt{3}} = 1252.25\text{ N}$

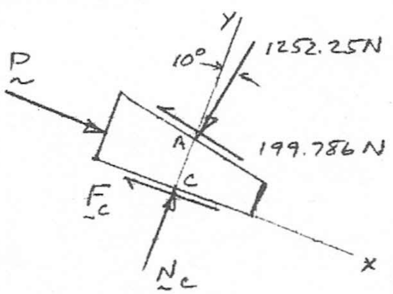
From (1), $F_A = F_B = 199.786\text{ N}$

$$\nearrow \Sigma F_y = 0: \quad N_C - (1252.25\text{ N})\cos 10° + 199.786\text{ N}\sin 10° = 0$$

$$N_C = 1198.53\text{ N}$$

**FBD Wedge:**

2




Impending slip $F_C = \mu_s N_C = 0.25(1198.53\text{ N}) = 299.63\text{ N}$

$$\searrow \Sigma F_x = 0: \quad P - 299.63\text{ N} - (199.786\text{ N})\cos 10°$$
$$-(1252.25\text{ N})\sin 10° = 0$$

**P** = 714 N $\measuredangle$ 20.0° ◀

5. a)

$v_0 = 6.0\,m/s$, $m = 1\,Kg$ and $y = 1\,m$

$(v_0)_y = v_0 \sin\theta = 6\sin\theta$

*- Vertical Motion: From O to A.

$v_y = (v_0)_y + at$ where $v_y$ at $A = 0$, $(v_0)_y = 6\sin\theta$ and $a = -g$

3 $\therefore\ 0 = 6\sin\theta - gt$, i.e. $t = \dfrac{6\sin\theta}{g}$

Now, $y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$

$1\,m = 0 + 6\sin\theta \times \dfrac{6\sin\theta}{g} - \dfrac{1}{2} g \times \left(\dfrac{6\sin\theta}{g}\right)^2$

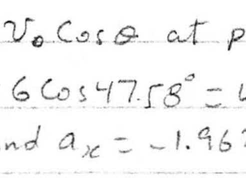
6 3 $1\,m = \dfrac{36\sin^2\theta}{g} - \dfrac{18\sin^2\theta}{g}$

$g = 18\sin^2\theta = 9.81$

$\therefore \sin^2\theta = \dfrac{9.81}{18} = 0.545$

$\sin\theta = 0.738$, $\therefore\ \theta = \underline{\underline{47.58^\circ}}$ ANS.

b) FBD - Brick




$N = W = 9.81\,N$

$\therefore F = \mu_k N = 0.2 \times 9.81 = 1.962\,N$

$F = ma_x$; $-1.962 = 1\,Kg\ a_x$

$\therefore\ a_x = -\dfrac{1.962}{1} = -1.962\,m/s^2$

6 $v^2 = v_0^2 + 2a_x(x - x_0)$, where $v = 0$, $(v_0)_x = v_0\cos\theta$ at point A

$= 6\cos 47.58^\circ = 4.05\,m/s$

and $a_x = -1.962\,m/s^2$

$0 = (4.05)^2 - 2 \times 1.962\,\Delta x$

$\therefore\ \Delta x = \dfrac{(4.05)^2}{2 \times 1.962} = 4.18\,m$ $\therefore$ Distance from A to B $= \underline{\underline{4.18\,m}}$ ANS

Now, $v = (v_0)_x + a_x t$, where $v = 0$, $v_0 = 4.05\,m/s$ & $a_x = -1.962\,m/s^2$

$0 = 4.05 - 1.962t$; $1.962t = 4.05$, $\therefore\ t = \dfrac{4.05}{1.962} = \underline{2.06\ sec.}$ ANS

END