STAT 241/251 MT2 sample

Disclaimer: These are just sample questions. Do not use this sample MT to assess the difficulty level of the actual MT. The actual midterm could be easier or harder than this sample. Do not use this sample to predict the topics on the MT. This sample is deliberately set longer so that I have a variety of questions that you can try out (it does not represent the length of your MT). Your midterm covers Ch 5, 6 and 7. Although I will not deliberate test you on earlier chapters prior to Ch 5, you have to understand that the material from Ch 3-7 are all connected. Therefore, it is possible to get a question on conditional probability, or you might be tested on your knowledge of exponential distributions if the question is on waiting time of a Poisson Process.

1. A machine is used to chop logs. The number of chops, X, before a log is split has distribution

$$P(X=r)=\begin{cases} p(1-p)^{r-1} & r=1,2,... \\ 0 & \text{otherwise} \end{cases}$$

   For an oak log, the value of p is $\frac{1}{3}$, but for a maple log, the value of p is $\frac{2}{3}$. In a large batch of logs, three-quarters of the logs are oak and the rest are maple. The random variable Y is the number of chops required to split a log chosen at random from this batch.

   (a) Show that $P(Y=r)=\frac{1}{6}(\frac{1}{3})^{r-1}+\frac{1}{4}(\frac{2}{3})^{r-1}$. [4 mark]

   (b) Find E(Y). [4 marks]

   (c) Determine P($Y \leq 4 \,|\, Y > 2$) [3 marks]

2. An advertising display contains a large number of light bulbs which are continually being switched on and off. Individual lights fail at random times, and each day the display is inspected and any failed lights are replaced. The number of lights that fail in any one-day period has a Poisson distribution with mean 2.2.

   (a) Calculate the probability that no light will need to be replaced on a particular day. [1 marks]

   (b) Calculate the probability that at least four lights will need to be replaced on a particular time. [2 marks]

   (c) Calculate the least number of consecutive days after which the probability of at least one light having to be replaced exceeds 0.9999. [3 marks]

   (d) Calculate the probability that, in a period of seven days, at least four lights will need to be replaced on at least two days. [3 marks] (Note: Just to be sure you understand what this is asking, here is another way to express this part. Let A be the event that at least four lights will need to be replaced in a particular day. You are to calculate that in a period of seven days, event A occurs on at least two days.)

3. The length of time for which an ordinary light-bulb will last may be taken to have a normal distribution with mean 600 hours and standard deviation 100 hours. The length of time for which a new long-life bulb will last may be taken to have a normal distribution with mean 2000 hours and standard deviation 200 hours.

   (a) One ordinary bulb is chosen at random. Find the probability that it will last more than 450 hours. [2 marks]

   (b) Two ordinary bulbs are chosen at random. Find the probability that the sum of the times for which they last will be less than 1100 hours. [2 marks]

   (c) One hundred ordinary bulbs are chosen at random. Find the probability that the mean of the times for which they last will be more than 595 hours. [3 marks]

   (d) One ordinary bulb and one long-life bulb are chosen at random. Find the probability that the long-life bulb lasts for more than three times as long as the ordinary bulb. [3 marks]

4. In each batch of manufactured articles, the proportion of defective articles is p. From each batch, a random sample of nine is taken and each of the nine articles is examined. If two or more of the nine articles are found to be defective, the batch is rejected; otherwise, it is accepted. Show that the probability that a batch is accepted is $(1-p)^8(1+8p)$. [3 marks]

   It is decided to modify the sampling scheme so that when one defective is found in the sample, a second sample of nine is taken (each of the nine articles is examined) and the batch rejected if this contains any defectives. With this exception, the original scheme is continued. Find an expression in terms of p for the probability that a batch is accepted. [3 marks].

   For this modified scheme, evaluate the average number sampled per manufactured batch over a large number of batches when p has the value 0.1. [3 marks] (Note: Just to be sure you understand what this is asking, you are asked to calculate the average number *of articles* sampled per manufactured batch over the long run.)