Solutions (Attached)

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

| Course | Number | Sections |
|---|---|---|
| Mathematics | 201 | All |
| **Examination** | **Date** | **Duration** |
| Midterm | 8 March, 2015 | 1 h 30 min |
| **Special Instructions:** | Only approved calculators are allowed **Show all your work for full marks.** | |

**1.** [9] (a) A line segment has the endpoints (-4,11) and (8,-5). Find the distance between the midpoint of the segment and the origin $(0,0)$.

(b) Find the equation of the line passing through the point $(4,0)$ and which is perpendicular to the line segment indicated in (**a**) above.

(c) Write the equation of the circle with the center at $(1,-3)$ and passing through the point $(4,0)$. (Hint: find first the radius of the circle.)

**2.** [12] Consider the quadratic function $f(x) = -x^2 + 4x + 5$.

(a) Express $f(x)$ in standard form.

(b) Find its vertex and indicate is it the maximum or the minimum of $f$.

(c) Find the $x$- and $y$-intercepts.

(d) Graph this function using the information you found above

**3.** [5] Consider the functions $f(x) = -2 + \sqrt{x^2+4}$ and $g(x) = \sqrt{x-3}$.

Find the domain and the range of $f$.

Find the function $f \circ g$ and determine its domain.

**4.** [12] Find the solutions of the following equations:

(a) $5^{2x} + 4 \cdot 5^x = 12$

(b) $\log_3(2x+1) - \log_3(x-1) = 2$

(c) $\log_2(4x) + \log_2(x-3) = 4$

**5.** [6] Find all horizontal and vertical asymptotes of $f(x) = \dfrac{2x^3 - 8x}{x(x^2+4x+3)}$.

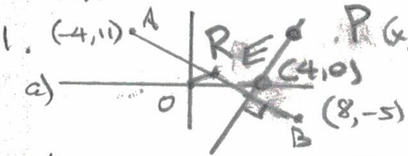
**6.** [6] Consider the function $f(x) = \dfrac{2x+1}{x+3}$.

(a) Find the inverse function $f^{-1}(x)$.

(b) Find the domain and the range of $f$, and the domain and range of$f^{-1}$.

**Bonus.** [3]: Assuming that today's world population is 7 billions and the continuous growth rate of the world population is 1.4% per year, estimate the world population expected in 15 years from now.

MAth 201 MidTerm March 2015

1\. a) 


Step1 Midpt AB $x = \frac{x_1+x_2}{2} = \frac{-4+8}{2} = \frac{4}{2} = 2$

$y = \frac{y_1+y_2}{2} = \frac{11+(-5)}{2} = \frac{6}{2} = 3$ ⇒ Mid pt R is pt (2,3)

Step2 distance $OR = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2-0)^2+(3-0)^2} = \sqrt{13}$

b) Eq. of line EP. Let P(x,y) be any pt on Required line through E(4,0)

Step1 slope AB

$= \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-5-11}{8-(-4)}$

$= \frac{-16}{12}$

$= -\frac{4}{3}$

Step3 line ⊥ to AB has Slope $-\frac{1}{-\frac{4}{3}} = \frac{3}{4}$

Step3 $M_{EP} = \frac{y_2-y_1}{x_2-x_1}$

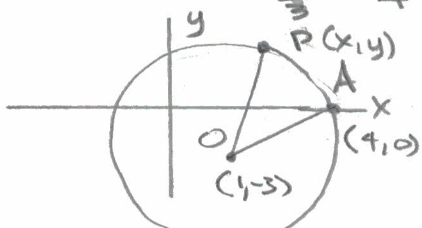
$\frac{3}{4} = \frac{y-0}{x-4}$

$4y = 3(x-4)$

$4y = 3x-12$

$y = \frac{3}{4}x - \frac{12}{4}$

$y = \frac{3}{4}x - 3$

c) 


Step1 Radius = distance OA

$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-1)^2+(0-[-3])^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

Step2 Eq. of circle let P(x,y) be any pt. on circle

$OR = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{18} = \sqrt{(x-1)^2+(y-[-3])^2}$

$18 = x^2-2x+1+y^2+6y+9$

OR WRITE: $x^2+y^2-2x+6y+9 = 0$

2\. a) Standard form means form where we get Turning pt (Complete Square)

$y = -x^2+4x+5$

$y = -(x^2-4x+\underline{4}) + 5 \underline{+4}$

$y = -(x-2)(x-2) + 9$

$y = -(x-2)^2+9$

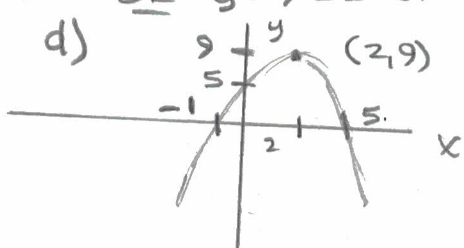
OR $y-9 = -(x-2)^2$

b) Vertex is (2,9)

opens down ⇒ Max At (2,9)

c)

| x int (let y=0) | | y int (let x=0) |
|---|---|---|
| $-x^2+4x+5=0$ | | $y=-x^2+4x+5$ |
| $x^2-4x-5=0$ | | $y=-0+0+5$ |
| $(x+1)(x-5)=0$ | | $y=5$ |
| $x+1=0$ | $x-5=0$ | |
| $x=-1$ | $x=5$ | |

d) 


3\. (i) $y = -2+\sqrt{x^2+4}$ Domain

$x^2+4 \geq 0$ this is true for all $x \in R$

⇒ Domain $x \in R$ or $(-\infty,\infty)$ or $-\infty < x < \infty$

Range: $y+2 = \sqrt{x^2+4}$ ; $\sqrt{x^2+4} > 0$

⇒ $y+2 > 0$ ⇒ $y > -2$

⇒ Range $y \in R \mid y > -2$

(ii) $(f \circ g)(x)$ means $f(g(x))$:

$f(x) = -2+\sqrt{x^2+4}$

$f(g(x)) = -2+\sqrt{[g(x)]^2+4}$

$f(g(x)) = -2+\sqrt{(\sqrt{x-3})^2+4}$

$f(g(x)) = -2\sqrt{x-3+4} = -2\sqrt{x+1}$

Domain $x+1 \geq 0 \Rightarrow x \geq -1$

⇒ Domain $x \in R \mid x \geq -1$.

4a) $5^{2x}+4(5^x)-12=0$

$(\cancel{5^x})^2+4(\cancel{5^x})-12=0$

$(5^x+6)(5^x-2)=0$

| | |
|---|---|
| $5^x+6=0$ | $5^x-2=0$ |
| $5^x=-6$ | $5^x=2$ |
| No x (why?) | $\log_e 5^x=\log_e 2$ |
| | $x\ln 5=\ln 2$ |
| | $x=\frac{\ln 2}{\ln 5}$ |

take log both sides

Basic log Rule ③ $\log_a M^P = p\log_a M$

Note

If Equation was:

$x^2+4x-12=0$ ① Quadratic

$(x+6)(x-2)=0$ Factor

| | |
|---|---|
| $x+6=0$ | $x-2=0$ |
| $x=-6$ | $x=2$ |

We have same Form

Except we have $\underline{5^x}$ instead of $\underline{x}$

4b) $\log_3(2x+1)-\log_3(x-1)=2$

$\log_3\frac{2x+1}{x-1}=2$

$3^2=\frac{2x+1}{x-1}$

$\frac{9}{1}=\frac{2x+1}{x-1}$

$9(x-1)=1(2x+1)$

$9x-9=2x+1$

$9x-2x=1+9$

$7x=10$

$x=\frac{10}{7}$

Basic log Rule ② $\log_a M-\log_a N = \log_a\frac{M}{N}$

Napier Notation

4c) $\log_2 4x+\log_2(x-3)=4$

$\log_2 4x(x-3)=4$

$2^4=4x(x-3)$

$16=4x^2-12x$

$4x^2-12x-16=0$

$x^2-3x-4=0$

$(x+1)(x-4)=0$

| | |
|---|---|
| $x+1=0$ | $x-4=0$ |
| $x=-1$ | $x=4$ |
| Discard (why?) | |

Basic log Rule ① $\log_a M+\log_a N=\log_a MN$

Napier

5. VA

$x(x^2+4x+3)=0$

$x(x+1)(x+3)=0$

| | | |
|---|---|---|
| $x=0$ | $x=-1$ | $x=-3$ |
| Num. $=0$ | Num $\neq 0$ | Num $\neq 0$ |
| No VA | $x=-1$ is VA | $x=-3$ is VA |

HA

$\lim_{x\to\infty}\frac{2x^3-8x}{x(x^2+4x+3)}=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{\frac{2x^3}{x^3}-\frac{8x}{x^3}}{\frac{x}{x}\frac{(x^2+4x+3)}{x^2}}$

$\lim_{x\to\infty}\frac{2-\frac{8}{x^2}}{1(1+\frac{4}{x}+\frac{3}{x^2})}$

$=\frac{2-0}{1(1+0+0)}$

$=\frac{2}{1}$

$=2$

$\Rightarrow$ HA: $y=2$

6. a) $y = \frac{2x+1}{x+3}$

$\frac{x}{1} = \frac{2y+1}{y+3}$

$x(y+3) = 1(2y+1)$

$xy + 3x = 2y + 1$

$xy - 2y = 1 - 3x$

$y(x-2) = 1 - 3x$

$y = \frac{1-3x}{x-2}$

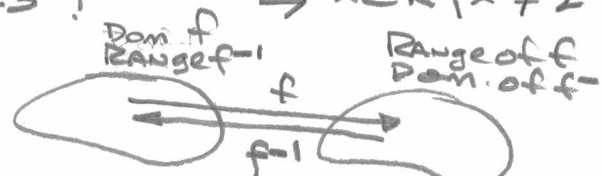
If $f(x) = \frac{2x+1}{x+3}$

then $f^{-1}(x) = \frac{1-3x}{x-2}$

b)

| $f(x)$ | $f^{-1}(x)$ |
|---|---|
| Dom: $x+3 \neq 0 \Rightarrow x \neq -3$ $\Rightarrow x \in R \mid x \neq -3$ | Dom: $x-2 \neq 0 \Rightarrow x \neq +2$ $\Rightarrow x \in R \mid x \neq 2$ |

c) For Ranges:




Since Range of $f$ = Domain $f^{-1}$
$= x \in R \mid x \neq 2$

Range of $f^{-1}$ = Domain $f$
$= x \in R \mid x \neq -3$

Bonus: Let W represent world population

$W = W_0 e^{rt}$

$W = 7(\text{billions})\, e^{.014(15)}$

$W = 8.64$ billions