

Instructor: Ş. Alaca

This test paper has 6 multiple choice questions and 2 long answer questions. It cannot be taken from the examination room. Only nonprogrammable calculators are allowed. Duration: 50 Minutes.

NAME :

STUDENT NO :

Multiple Choice Questions. Circle the correct answer. Three marks each.

1. Which of the following sets are subspaces of R^3 .

$$U = \left\{ \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \mid a - b = 1 \right\}, \quad V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a + b = c \right\},$$

$$W = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}, \quad H = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

a) U and V only b) V and W only c) V and H only d) U and W only

2. If $\det \begin{pmatrix} a & b & c \\ x & y & z \\ u & v & w \end{pmatrix} = 4$, what is $\det \begin{pmatrix} 3a & 3b & 3c \\ u - 2x & v - 2y & w - 2z \\ x & y & z \end{pmatrix}$?

a) 12 b) -12 c) 24 d) -24

3. Let A and B be 3×3 matrices such that $\det A = 8$ and $\det B = 4$. What is $\det(2A^T B^{-1})$?

a) 32 b) 16 c) 8 d) 4

4. Let S be the parallelogram determined by the vectors $u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

What is the area of S ?

- a) 32 unit^2 b) 16 unit^2 c) 8 unit^2 d) 4 unit^2

5. Let A be a 6×7 matrix such that $\text{rank} A = 4$, i.e, dimension of $\text{Col} A = 4$.

What is dimension of $\text{Nul} A$?

- a) 4 b) 3 c) 2 d) 1

6. Let H be a subspace of R^4 and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$ be a basis for H .

If $[y]_{\mathcal{B}} = \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}$, what is the vector y ?

- a) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ b) $\begin{bmatrix} 4 \\ 5 \\ -6 \\ 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 \\ 4 \\ 5 \\ -6 \end{bmatrix}$ d) $\begin{bmatrix} -2 \\ 7 \\ 6 \\ 5 \end{bmatrix}$

Long answer questions. Show all your work.

[14] 1. You are given that $A = \begin{bmatrix} 1 & 3 & 2 & 1 & 3 \\ 2 & 7 & 5 & 1 & 8 \\ 1 & 3 & 2 & 2 & 4 \\ 2 & 9 & 7 & -1 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 & 1 & 3 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a) Find a basis for $\text{Col}A$.

b) Find a basis for $\text{Nul}A$.

c) Find the dimension of $\text{Col}A$ and the dimension of $\text{Nul}A$.

d) Determine if the vector $\begin{bmatrix} 4 \\ 6 \\ 7 \\ 2 \end{bmatrix}$ is in the column space of A . Explain your reason clearly.

[8] 2. Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}$, $b = \begin{bmatrix} -2 \\ 1 \\ 3 \\ 3 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$.

You are given that $\det A = -3$.

Use Cramer's Rule to find x_4 in the matrix equation $Ax = b$.