CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Duration**
Midterm Test	10 March 2019	1 h 30 min
Special Instructions:	Only approved calculators are allowed **Show all your work for full marks**	

1. (12 marks): **(a)** Solve for x : $2\log_2(x) = 2 + \log_2(x+3)$.

(b) Let $f(x) = \log_2(3-x)$ and $g(x) = 4x^2 - 1$. Find the composite function $f \circ g$ and determine its domain and its range.

(c) Let $f(x) = 3^{x^4-1}$ and $g(x) = 3^{5x-1}$. Determine which of these functions **is not** invertible and which one is (**explain!**), and find the inverse of the invertible function.

2. (6 marks) Find **(a)** all horizontal and **(b)** all vertical asymptotes of the graph

$$y = \frac{|x|\sqrt{9x^4+6x^2+2}}{(2x+3)(x+1)^2}$$

3. (8 marks) Find the limit or explain why the limit does not exist:

(a) $\lim\limits_{x\to\infty} \left(\sqrt{x^2-2x} - x\right)$

(b) $\lim\limits_{x\to 0} \dfrac{\sqrt{2x^2+5x^4}}{x}$

4. (4+3 marks) Given the function $f(x) = \dfrac{2}{x-3}$,

(a) Use the definition of derivative to calculate the derivative $f'(x)$.

(b) Write equation of the tangent line to the curve $y = f(x)$ at the point $(1, -1)$.

(continued on the other side)

5. (5 marks) Find the third derivative of $f(x) = e^{bx}\,(e^{bx} + e^{3-bx})$, where b is a parameter, and calculate its **exact** value at $x = 0$, i.e. calculate $f'''(0)$.
(HINT: simplify $f(x)$ before calculating the derivatives.)

6. (12 marks) Find the derivatives of the following functions:

(**a**) $f(x) = \dfrac{2x^3 + x - 10}{x\sqrt{x}}$

(**b**) $f(x) = e^x + x^e - e\,x$

(**c**) $f(x) = \dfrac{\tan(3x)}{1 + x^2}$

(**d**) $f(x) = \cos^2\left(\sin(3x) + x^3\,e^x\right)$

Bonus Question (3 marks). Consider the function

$$f(x) = \begin{cases} x + a & \text{if } x \le 1 \\ ax^2 + b & \text{if } x > 1 \end{cases}$$

where a and b are parameters. Find the values of a and b that make $f(x)$ differentiable everywhere, or explain why this is impossible.