FINE 342: Corporate Finance
Fall 2015
Vadim di Pietro

Assignment 1

- The assignment is **due in class on Tuesday Oct 27**. Solutions will be posted shortly after the due date so late assignments cannot be accepted.
- You may work in groups of up to 5 students. Only 1 copy needs to be handed in. Typed or handwritten assignments are both acceptable.

Private Equity Issuance

1) Today is t = 0. Vapepsi is a private company that produces high end luxury diet soda. The company's founder, Vadim, is seeking round 1 financing from 342VC, Montreal's top Venture Capital firm.

The company has existing debt with $3M face value (0-coupon) and maturing at t = 3.

The company is seeking equity financing to expand operations. In order to begin small scale production (phase 1), Vapepsi needs to raise $1M at t = 0. Phase 1 will last 2 years and there is a 70% chance phase 1 will be successful.

If phase 1 is successful, the company will proceed to phase 2a, which would require another round of equity financing of $3M at t = 2. After phase 2a, the company has a 50% chance of being worth $22M at t = 3 and a 50% chance of being worth $6.6M at t = 3.

If phase 1 is not successful, the company will proceed to phase 2b, which would require another round of equity financing of $1M at t = 2 to continue small scale production. After phase 2b, the company has a 50% chance of being worth $6.6M at t = 3 and a 50% chance of being worth $2.2M at t = 3.

Note that Phase 2 financing will go through Sequoia Capital.

Assume that investors are risk neutral and that the appropriate discount rate for all expected cash flows is the risk free rate of 10%.

a) What fractional ownership of Vapepsi will 342VC require in order to raise $1M at t = 0?

$$1 = x\frac{0.7\left(0.5\left(\frac{22-3}{1.10}\right)+0.5\left(\frac{6.6-3}{1.10}\right)-3\right)+0.3\left(0.5\left(\frac{6.6-3}{1.10}\right)+0.5\left(\frac{0}{1.10}\right)-1\right)}{1.10^2}$$

→ x = 22.9%

Note:

$\left[0.5\left(\frac{22-3}{1.10}\right)+0.5\left(\frac{6.6-3}{1.10}\right)-3\right]$ is the premoney value at t = 2 if phase 1 is successful

$\left[0.5\left(\frac{6.6-3}{1.10}\right)+0.5\left(\frac{0}{1.10}\right)-1\right]$ is the premoney value at t = 2 if phase 1 is unsuccessful

Alternatively, you could also solve this from a "post-money" point of view. However, in this case you would have to take into account that after equity issuance at t = 2 the fractional ownership is no longer x. You would have to solve parts b and c first to know how much fractional ownership is being given up in round 2 financing. In the equation below x is the fractional ownership the company is giving up at t = 0, $x(1-0.292)$ is the fractional ownership of stage one investors after stage two financing has occurred in the good state and $x(1-0.611)$ is the fractional ownership of stage one investors after stage 2 financing in the bad state.

$$1 = 0.7x(1-0.292)\frac{\left(0.5\left(\frac{22-3}{1.10}\right)+0.5\left(\frac{6.6-3}{1.10}\right)\right)}{1.10^2} + 0.3x(1-0.611)\frac{\left(0.5\left(\frac{6.6-3}{1.10}\right)+0.5\left(\frac{0}{1.10}\right)\right)}{1.10^2}$$

→ x = 22.9%

Note:
$\left[0.5\left(\frac{22-3}{1.10}\right)+0.5\left(\frac{6.6-3}{1.10}\right)\right]$ **is the postmoney value at t = 2 if phase 1 is successful**
$\left[0.5\left(\frac{6.6-3}{1.10}\right)+0.5\left(\frac{0}{1.10}\right)\right]$ **is the postmoney value at t = 2 if phase 1 is unsuccessful**

b) What fractional ownership of Vapepsi would Sequoia Capital require to proceed with Phase 2a financing?

$$3 = y_{2a}\left[0.5\left(\frac{22-3}{1.10}\right)+0.5\left(\frac{6.6-3}{1.10}\right)\right]$$

→ $y_{2a} = 29.2\%$

c) What fractional ownership of Vapepsi would Sequoia Capital require to proceed with Phase 2b financing?

$$1 = y_{2b}\left[0.5\left(\frac{6.6-3}{1.10}\right)+0.5\left(\frac{0}{1.10}\right)\right]$$

→ $y_{2b} = 61.1\%$

d) Assume that at t = 0 there are 100 shares outstanding before the private equity issuance. How many shares will be issued at t = 0? (Fractional units are always OK.)

$$0.229 = \frac{N_1}{100+N_1}$$

→ N_1 **= 29.7**

(although not asked here, the issue price is 1M/29.7 = 33,651.39)

e) How many shares would be issued at t = 2 to fund Phase 2a? At what price?

$$0.292 = \frac{N_{2a}}{129.7 + N_{2a}}$$

➔ N_{2a} = 53.5

➔ $P_{2a} = \frac{3M}{53.5} = 56,066.34$

f) How many shares would be issued at t = 2 to fund Phase 2b? At what price?

$$0.611 = \frac{N_{2b}}{129.7 + N_{2b}}$$

➔ N_{2b} = 203.8

➔ $P_{2b} = \frac{1M}{203.8} = 4,905.81$

g) List the percentage ownership of the company for each owner at

i) t = 0 before issuance

Founder owns 100%

ii) t = 0 after Phase 1 issuance until t = 2 before Phase 2 issuance

342VC owns 22.9%
Founder owns 77.1%

iii) t = 2 after Phase 2 issuance for case 2a and case 2b:

case 2a:

Sequoia owns 29.2%
342VC owns 22.9%(1-0.292) = 16.2%
Founder owns 77.1%(1-0.292) = 54.6%

case 2b:

Sequoia owns 61.1%
342VC owns 22.9%(1-0.611) = 8.9%
Founder owns 77.1%(1-0.611) = 30.0%

h) What is the value of total equity at t = 0 before issuance?

1M/22.9% - 1M = 3,365,139

i) What is the fair value per share at t = 0 before issuance? How does this compare to the t = 0 issue price?

3,365,139/100 = 33,651.39 (same as issue price (see part d))

j) What is the fair value of debt at t = 0?

$$B = \frac{0.7\left(0.5(3M) + 0.5(3M)\right) + 0.3\left(0.5(3M) + 0.5(2.2M)\right)}{1.10^3} = 2.16M$$

IPO

2) Today is t = 0. After many years of wildly successful private ownership, Vapepsi is ready to go public. The company is seeking to raise $50M in an IPO to expand operations internationally.

There is a 50% chance the expansion will be very successful, in which case the company will be worth $195M at t = 1. There is a 50% chance the expansion will be moderately successful, in which case the company will be worth $135M at t = 1.

At t = 0 there are 1M shares outstanding and no debt. How many shares need to be issued, and at what price, in order to raise the required $50M? Ignore any underwriting costs.

$$50 = \frac{N}{1{,}000{,}000 + N}\left[0.5\left(\frac{135}{1.1}\right) + 0.5\left(\frac{195}{1.1}\right)\right]$$

- **N = 500,000**
- **P = 50M/500,000 = 100**

3) Repeat question 2, but under the assumption that the company has pre-existing debt with FV = $10M, 0-coupon, and maturing at t = 1. Before answering numerically, think about whether the number of shares should be higher/lower and if the issue price should be higher or lower compared to your answers in number 2. Also, what is the price of the bond at t = 0?

$$50 = \frac{N}{1{,}000{,}000 + N}\left[0.5\left(\frac{135 - 10}{1.1}\right) + 0.5\left(\frac{195 - 10}{1.1}\right)\right]$$

- **N = 550,000**
- **P = 50M/550,000 = 90.91**

(not surprisingly you have to issue more shares at a lower price (give up a larger fraction of the company), compared to Q2, since in Q3 the fact that there is debt makes the equity less valuable.)

IPO and convertible bonds

4) A convertible bond gives the bond owner the right, but not the obligation, to convert the bond into a predetermined number of shares at a certain date(s). Repeat question 2, but under the assumption that the company has a pre-existing convertible bond outstanding (and no other debt). If the bond is not converted the payout to bondholders is $10M at t = 1. If the bond is converted, the company has to issue and give 100,000 shares to the bondholders, in exchange for the bond (which means the company won't have to pay out the $10M face value).

You may assume that if the expansion is successful, the bond will be converted at t = 1. If the expansion is not successful, the bond will not be converted.

a) Based on the information above, what would the IPO issue price be, and how many shares would have to be issued in the IPO? (Hint: you may need to use Excel Solver to solve your equation.) Before you answer numerically, how do you think your answers will compare to those in number 3?

$$50 = 0.5\frac{N}{1,000,000+N}\left[\frac{135-10}{1.1}\right]$$
$$+0.5\frac{N}{1,000,000+N+100,000}\left[\frac{195}{1.1}\right]$$

- N = 554,947.12
- P = 50M/554,947.12 = 90.10

(not surprisingly you have to issue more shares at a lower price (give up a larger fraction of the company), compared to Q3, since the value of the convertible bond is greater than the value of the straight bond. In the good state equity holders are "paying" more than $10M by giving up a fraction of their ownership in exchange for the bond.)

b) What is the value of the convertible bond at t = 0?

$$B = 0.5\left[\frac{10}{1.1}\right] + 0.5\frac{100,000}{1,000,000+554,947.12+100,000}\left[\frac{195}{1.1}\right] = 9,901,297.21$$

(Note: this is greater than the value of a straight bond, which would be 10M/1.1 = 9.09M.)

c) Confirm that it is only optimal for the bondholders to convert in the good state.

Good state t = 1:

$$\frac{100,000}{1,000,000+554,947.12+100,000}[195] = 11,782,853.86 > 10,000,000$$

So converting is optimal.

Bad state t = 1:

$$\frac{100,000}{1,000,000+554,947.12+100,000}[135] = 8,157,360.36 < 10,000,000$$

So not converting is optimal.

d) Suppose there were greater uncertainty (but same expected value) over the value of the company after expansion (for example, instead of $195M and $135M, suppose the possible values were $250M and $80M). (Assume we can still use the 10% discount rate.) Would the issue price you found in part a still represent a fair investment for the IPO investors? Explain intuitively. No calculations are required.

No it would not. In the good state the equity holders are giving up more than compared to part a. Thus the issue price would have to be lower. Another way to think about this is that higher volatility increases the value of the convertible bond in this example, which decreases the value of equity (since the value of assets is equal to equity + convertible bond, and the value of the assets has not changed).

SEO and share rights

5) After several years of exceptional operations as a public company, Vapepsi wants to expand operations to Emerging Markets. In order to finance these operations, the company will proceed with a SEO. The company's current share price is $20 (assume this is the fair price as well) and after several stock splits there are now 100M shares outstanding. The company's CFO, Elmo, suggests issuing 20M shares at a price of $10 each. In particular, Elmo suggests the entire offering should be allocated to Scam Capital, a small Hedge Fund that has expressed an interest in taking a stake in Vapepsi.

a) If this SEO structure were to be announced, what would the share price of Vapepsi adjust to right after the announcement?

$$\frac{20(100M) + 10(20M)}{120M} = 18.33$$

(Note: we are assuming that the expansion into EM has a 0 NPV overall, or if it did have an NPV, that was already taken into account in the original price of $20 per share.)

b) How much profit would Scam Capital make?

$$20M(18.33 - 10) = 166.67M$$

c) How much would current shareholders lose in total?

The hedge fund's gain is the current shareholders loss: 166.67M

This can also be seen as

$$100M(20 - 18.33) = 166.67M$$

(Note: we are assuming that the expansion into EM has a 0 NPV, or if it did have an NPV, that was already taken into account in the original price of $20 per share.)

The company's CEO, Vadim, realizes that Elmo's SEO proposal makes no sense. Moreover, it turns out that the Fund Manager at Scam Capital, Zoe, is Elmo's best friend. The two were trying to pull off a scam. Vadim quickly fires Elmo and decides to proceed with an SEO for 20M shares, priced at $10, but this time including a rights issue, where each current shareholder gets 1 right per share.

d) What is the stock price reaction after the announcement of the SEO with rights issue? (Assume that the announcement itself has no informational content about the future prospects of the company.)

There is no stock price reaction following the announcement, it remains at $20. Underpricing the SEO is bad for current shareholders, but this is exactly offset by the fact that the existing shareholders have the right to benefit by buying the underpriced SEO (or selling their rights). The stock price will only drop on the ex-rights date.

e) How much is 1 right worth?

There are 100M rights and 20M shares to be issued. It thus takes 100M/20M = 5 rights to buy 1 share in the SEO. The share price will eventually drop to 18.33 on the ex-rights date. Being able to buy a security that is worth 18.33 for a price of 10 has a value of 8.33 per share. But since it takes 5 rights to buy 1 share, the value per right is 8.33/5 = 1.67

(Note that 18.33 + 1.67 = 20. So the SEO with a rights offering does not affect shareholder wealth.)

f) Suppose Vadim previously owned 40% of the company. What fraction of the company will he own if he fully exercises his rights? (Assume all rights are eventually exercised.)

Vadim owned 40M shares. He got 40M rights, which allows him to buy 8M shares. He now has 48M shares out of a total of 120M shares, so his ownership is still 48M/120M = 40% of the company.

g) Suppose Vadim previously owned 40% of the company. What fraction of the company will he own if he sells all his rights? (Assume all rights are eventually exercised.)

In this case Vadim still owns 40M shares, out of 120M, which is 40M/120M = 33.33%

Debt priority

6) After dominating the soda market for 10 years, Vapepsi is now the largest company in the world, with a market capitalization of $1tr. The company's CEO, Vadim, is now looking to acquire Coca-Cola, Apple, Google, and Facebook. People in Vadim's entourage are now concerned his rational thought is being affected by late stage aspartame poisoning. Nevertheless, the acquisitions are to be financed by the company's first ever debt issuance. The company plans to issue 5 classes of debt:

Tranche S1: Senior Secured, Face Value = 200bn, Secured by Asset A
Tranche J1: Junior Secured, Face Value = 100bn, Secured by Asset A
Tranche J2: Junior Secured, Face Value = 50bn, Secured by Asset B, (ranks most senior with respect to Asset B since it is secured by Asset B, but ranks same as Tranche J1 with respect to unpledged assets).
Tranche D: Unsecured Debt, Face Value = 200bn (same seniority as J1 and J2 with respect to unpledged assets)
Tranche E: Subordinated Debt (lowest seniority)

In the extremely unlikely event that Vapepsi goes bankrupt and is liquidated assume:

Scenario 1: value of Asset A = 250bn, value of Asset B = 40bn, value of unpledged assets = 100bn.
Scenario 2: value of Asset A = 10bn, value of Asset B = 40bn, value of unpledged assets = 100bn.

For both scenarios, describe how much each category of debtholders recover? (neglect due interests)

Scenario 1:

S1: gets 200 from Asset A. So S1 is paid in full: 200bn.

J1: Asset A has 50 value left, which goes to J1, and J1 still has a claim of 50 on unpledged assets.

J2: gets 40 from Asset B and has a claim of 10 on the unpledged assets.

D: has a claim of 200 on unpledged assets

The total claims of J1, J2 and D on the unpledged assets is 50 + 10 + 200 = 260. This is greater than the remaining 100 worth of unpledged assets. So the 100 of unpledged assets is distributed to J1, J2, and D on a pro rata basis.

J1: J1 gets 50 + (50/260)(100) = 69.23bn

J2: J2 gets 40 + (10/260)(100) = 43.85bn

D: D gets (200/260)(100) = 76.92bn

E: gets 0

Note: the total amount recovered (390) is equal to the total value of assets (390).

Scenario 2:

S1: gets 10 from Asset A, and is still owed 200 - 10 = 190, but can only recover 100 from the unpledged assets. So S1 gets 110bn.

J1: Asset A has 0 value left, and there are no unpledged assets left, so J1 gets 0.

J2: gets 40 from Asset B and has a claim of 10 on the unpledged assets, but there are no unpledged assets left, so B gets 40.

D and E get 0.

Note: the total amount recovered (150) is equal to the total value of assets (150).

7) Two firms F_u and F_l hold identical assets that generate earnings before interest every year in perpetuity, that depend on the economy, as shown in the table below. F_u has 2,000 shares outstanding and F_l has 200 shares outstanding. F_u is all equity (unlevered). F_l has outstanding perpetual debt with face value $500,000 and a coupon rate of 5%. The risk-free rate is r_f = 4%. The share price for F_U is $1,000.

State of the Economy	Earnings bf interests
Recession	$30,000
Normal	$80,000
Boom	$120,000

a) Assuming no frictions, what is the share price for the levered company?
(Note: you do not know the probability of each state, and you do not know the appropriate discount rate for risky cash flows. Still, there is a way to find the share price of the levered company using replication/arbitrage arguments. Specifically, show how you could create a synthetic share in the levered company by buying a certain number of shares in the unlevered company, together with personal borrowing in perpetuity of a certain amount. The share price of the levered company will equal the cost of the replicating portfolio.)

Value of leveraged company debt (500,000*5%)/4% = 625,000
To replicate 1 share of the leveraged company (which represents 1/200 = 0.5%, you need to buy 0.5%, which is (0.5%)(2,000) = 10 shares of the unleveraged company and borrow 0.5%(625,000) = 3,125 at the risk free rate of 4% in perpetuity.

The cost of this replicating strategy is 10(1,000) – 3,125 = 6,875, which much be the value of one share in the leveraged company.

Note: let's confirm the replicating strategy actually works:

One share in the leveraged company gives cash flows in each state equal to
(30,000 – (5%)(500,000))(1/200) = 25
(80,000 – (5%)(500,000))(1/200) = 275
(120,000 – (5%)(500,000))(1/200) = 245

And the cash flows in the replicating strategy give in each state:
(30,000)(10/2000) – 3,125(4%) = 25
(80,000)(10/2000) – 3,125(4%) = 275
(120,000)(10/2000) – 3,125(4%) = 245

b) Assume the share price for the levered company was $1 greater than your answer to part a. Describe how you would construct an arbitrage. Assume you are shorting one share of the levered company. Specify how many shares you are buying/shorting of the unlevered company, and how much money you are borrowing/lending in perpetuity.

You short one share of the leveraged company, buy 10 shares of the unleveraged company and borrow 3,125 at the risk free rate of 4% in perpetuity. Your net cash flow at t = 0 is a profit of $1 and the cash flows for t = 1 to infinity will cancel out. (Recall, when you are short a stock, you have to pay the dividends.)

8) Topic: Valuation in frictionless markets - multiple divisions

(See solutions to Assignment 2, Fall 2014)

Today is t = 0. You were recently appointed the CFO of a firm with two assets. Asset 1 is a factory that produces phones. Asset 2 is a factory that produces micro-chips. The market value of your firm's debt is $100M and the market value of your firm's equity is $100M. The beta of your firm's equity is 2, and your firm's debt is risk free. The expected return on the market portfolio is 10% and the risk free rate is 2%. Assume the CAPM holds. Assume frictionless markets.

a) What is the asset beta of your firm?

You expect Asset 1 will generate cash flows of $10M a year, starting at t = 1, in perpetuity. You also know that firm X, which only produces phones (and thus has the same asset beta as your Asset 1) has the following characteristics: Equity Beta = 1, Debt Beta = 0.1, and Debt-to-Equity ratio = 0.5.

b) What is the value of Asset 1? (Hint: use the information for company X to back out the asset beta for firm X, which corresponds to the asset beta of Asset 1. Use this Beta together with the CAPM formula to obtain the appropriate discount rate for Asset 1 cash flows.)

c) What is the value of Asset 2? (Hint: Value of firm = Value of Equity + Value of Debt = Value of Asset 1 + Value of Asset 2.)

d) What is the Beta of Asset 2? (Hint: Beta of total Assets (your answer to part a), is a weighted average of Beta Asset 1 and Beta Asset 2. This will allow you to back out the Beta of Asset 2).

Engineers in the micro-chip factory (Asset 2) now discover an opportunity to invest in a new production technology that would enable it to produce better micro-chips. The required investment would be $15M at t = 0, and would increase expected Asset 2 cash flows by $4M per year, starting at t = 1, in perpetuity. Assume the systematic risk (the asset beta) of Asset 2 will be unaffected by the new production technology.

e) Would you recommend investing in the new production technology? (Hint: the appropriate discount rate for asset 2 cash flows depends on Asset 2 beta and the CAPM formula. Calculate the NPV of the proposed project using this discount rate.)

9) Topic: Valuation in frictionless markets – share buyback

(See solutions to Assignment 2, Fall 2014)

BuyBack342 has no outstanding debt. The current stock price is $20, and there are 3M shares outstanding. The company pays out all earnings as dividends, and does not expect to grow. The current stock Beta is 1. The expected return on the market is 10%. The risk free rate is 5%. Assume the CAPM holds. Assume frictionless markets.

a) What is the expected return on the stock?

b) What are the expected earnings (and therefore dividends) per share?

c) What are the company's expected total earnings per year?

d) What is the company's PE ratio at t = 0? (E refers to expected earnings per share at t = 1, and P is current price.)

Suppose the company issues $40M (present value) of risk free perpetual debt, and uses that money to repurchase, and retire shares.

e) How many shares does the company repurchase, and at what price?

f) After shares are repurchased: What is the stock beta?

g) After shares are repurchased: What is the required return on equity (cost of equity)?

h) After shares are repurchased: What is the expected earnings per share?

i) After shares are repurchased: Verify that the stock price you found in part e is correct by discounting the perpetual cash flows to equity (per share) at the new cost of equity

j) After shares are repurchased: What is the PE ratio (same as defined above)?

k) Will shareholder wealth increase, decrease, or remain the same as a result of the buyback?