1) $f(0) = 4$, $f(1) = -1$ } since the function is changing its sign, then it has a root between $[0,1]$.

* when $f(x) = 0 \Rightarrow x^3 - 6x + 4 = 0$.
then $x = \frac{x^3+4}{6} = \frac{x^3}{6} + \frac{2}{3} \Rightarrow \boxed{g(x) = \frac{x^3}{6} + \frac{2}{3}}$

$\Rightarrow |g'(x)| = \left|\frac{3x^2}{6}\right| = \left|\frac{x^2}{2}\right| = \frac{x^2}{2} \leq \frac{1}{2} < 1$ in $[0,1]$.

* $g(x)$ will satisfy the condition.

* $x_{n+1} = g(x_n)$ with $x_0 = 0.5$.

$g(x_0) = 0.687500 = x_1$.
$g(x_1) = 0.720826 = x_2$
$g(x_2) = 0.729089 = x_3$
$g(x_3) = 0.731261 = x_4$
$g(x_4) = 0.731840 = x_5$.
$g(x_5) = 0.731995 = x_6$.
$g(x_6) = 0.732036 = x_7$
$g(x_7) = 0.732047 = x_8$
$g(x_8) = 0.732050 = x_9$
$g(x_9) = 0.732050 = x_{10}$
$g(x_{10}) = 0.732051 = x_{11}$ } $\Rightarrow$ the root is $\boxed{0.732051}$
$g(x_{11}) = 0.732051$

1

2) $\begin{cases} \ln x = 9 - x^2 \\ \text{four decimal places} \\ x_0 = 2 \end{cases}$

* $\ln x = 9 - x^2 \Rightarrow f(x) = x^2 + \ln x - 9 = 0$.

$$f'(x) = 2x + \frac{1}{x}.$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{-4.3069}{\frac{9}{2}} = 2.9571$$

$$x_2 = 2.957 - \frac{0.8286}{6.2524} = 2.8246$$

$$x_3 = 2.8246 - 0.0028 = 2.8218$$

$$x_4 = 2.8218 - \frac{-0.0001}{5.9980} = 2.8218$$

$\Rightarrow$ the solution $\boxed{x = 2.8218}$

we can verify:

$$\ln(2.8218) - 9 + (2.8218)^2 = 0$$

$$0 = 0 \checkmark$$

3) a) $b = 0$ and $r = \frac{2}{3}$.

We have $a = \frac{b}{1-r} = 0$. and; $|r| < 1 \Rightarrow$ stable

* solution: $a_k = r^k(a_0 - a) + a$

$$= \left(\frac{2}{3}\right)^k 6 = \frac{2^k}{3^k} . 2 . 3 = \frac{2^{k+1}}{3^{k-1}}$$

b) $b = 100$ and $r = \frac{4}{5}$.

We have $a = \frac{b}{1-r} = \frac{100}{1-\frac{4}{5}} = 500$; $|r| < 1 \Rightarrow$ stable.

* solution: $a_k = r^k(a_0 - a) + a$

$$= \left(\frac{4}{5}\right)^k (20 - 500) + 500 = -480\left(\frac{4}{5}\right)^k + 500$$

c) $b = 500$ and $r = \frac{-6}{5}$

We have $a = \frac{b}{1-r} = \frac{500}{\frac{5}{5}+\frac{6}{5}} = \frac{2500}{11}$; $|r| > 1 \Rightarrow$ Unstable

* solution: $a_k = r^k(a_0 - a) + a$

$$= \left(\frac{-6}{5}\right)^k \left(25 - \frac{2500}{11}\right) + \frac{2500}{11} = -\frac{2225}{11} . \left(\frac{6}{5}\right)^k + \frac{2500}{11}$$

d) $b = -30$ and $r = 2$

We have $a = \frac{b}{1-r} = \frac{-30}{-1} = 30$; $|r| > 1 \Rightarrow$ unstable

* solution: $a_k = r^k(a_0 - a) + a$

$$= 2^k(0 - 30) + 30 = -30 . 2^k + 30 = 30(1 - 2^k).$$

4) a) * This is a predator-prey interaction.

* $\underline{M_{n+1} = 1.3 M_n - 0.002 O_n M_n}$ : Mice (prey).

$\underline{1.2}$ : growth of mice population due to (reproduction)

$\underline{-0.001}$ : negative sign means the population of Mice deacrease with the presence of owls.

* $\underline{O_{n+1} = 0.6 O_n + 0.0004 O_n M_n}$ : owls (predator).

$\underline{0.6}$ : Owl population decrease if no Mice or Natural death

$\underline{0.0004}$ : represent the interaction between owls and Mice / owl population increase with the presence of Mice ...

b) Equilibrium is when the population of both species remain the same overtime: $M_{n+1} = M_n$ and $O_{n+1} = O_n$.

* $\overbrace{M_{n+1}}^{=M_n} = 1.3 M_n - 0.002 O_n M_n$

$\Rightarrow -0.3 M_n = -0.002 O_n M_n \Rightarrow \boxed{O_n = 150}$

* $\overbrace{O_{n+1}}^{=O_n} = 0.6 O_n + 0.0004 O_n M_n$.

$O_n = 0.6 O_n + 0.0004 O_n M_n \Rightarrow 0.4 \cancel{O_n} = 0.0004 \cancel{O_n} M_n$

$\Rightarrow \boxed{M_n = 1000}$

c) $M_0 = 1200$ and $O_0 = 100$.

$\underline{n=0}$: $M_1 = 1.3M_0 - 0.002O_0M_0 = 1320$

$O_1 = 0.6O_0 + 0.0004O_0M_0 = 108$

$\underline{n=1}$: $M_2 = 1.3M_1 - 0.002O_1M_1 = 1430.88$

$O_2 = 0.6O_1 + 0.0004O_1M_1 = 121.824$

$\underline{n=2}$: $M_3 = 1.3M_2 - 0.002O_2M_2 = 1511.51295$

$O_3 = 0.6O_2 + 0.0004O_2M_2 = 142.82061$

$\underline{n=3}$: $M_4 = 1.3M_3 - 0.002O_3M_3 = 1533.216$

$O_4 = 0.6O_3 + 0.0004O_3M_3 = 172.1042$

$\underline{n=4}$: $M_5 = 1465.34$

$O_5 = 208.7$

$\underline{n=5}$: $M_6 = 1293.009$

$O_6 = 246.6$

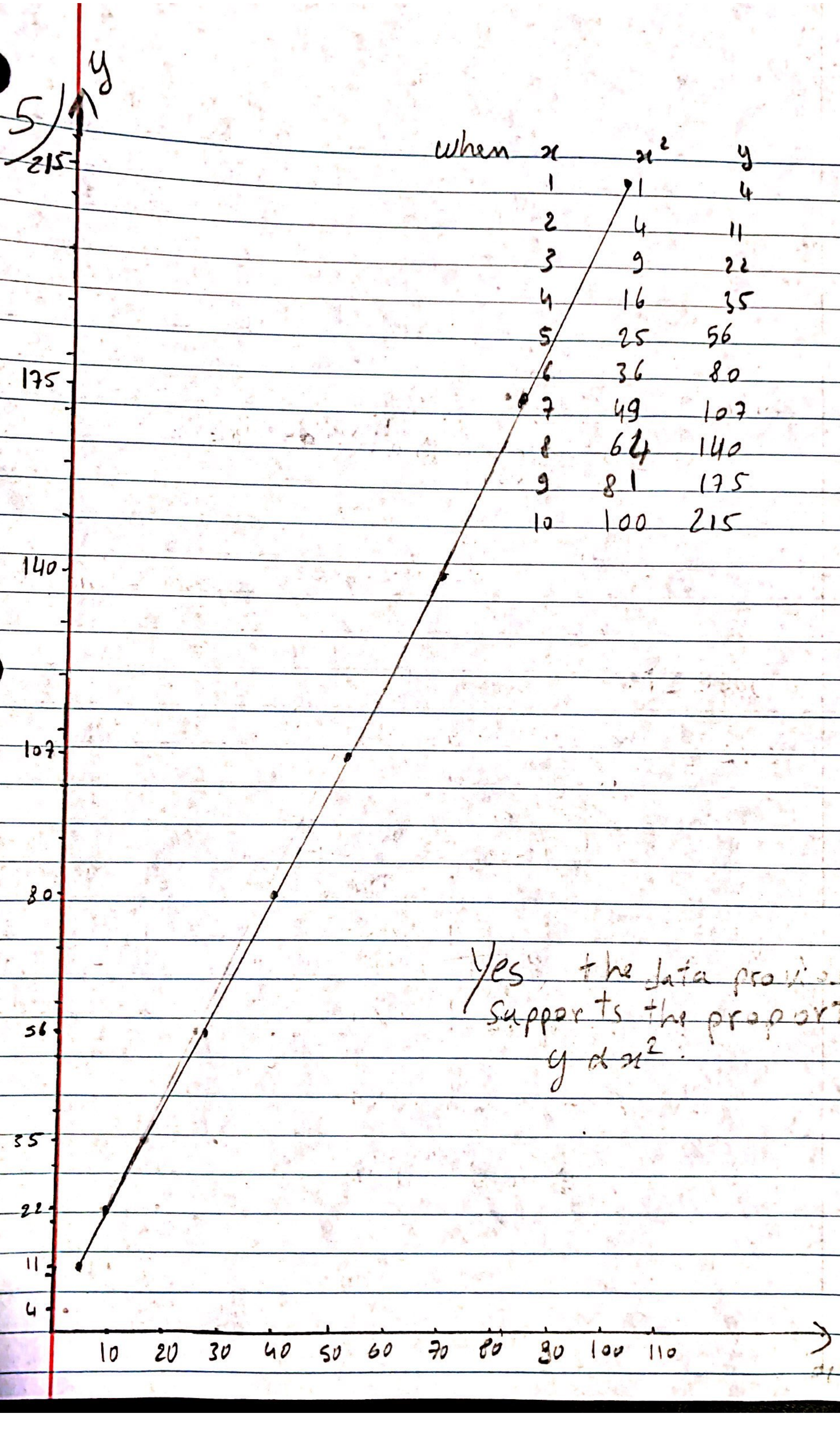
$\underline{n=6}$: $M_7 = 1044$

$O_7 = 274.8$

$\underline{n=7}$: $M_8 =$

It looks like the population of of Mice is decreasing while the population of the Owls is increasing when n increase.
If we run these function on Matlab, I predict that the when the number of Mice will go towards zero the population of Owls will decrease, as it's their primary source of food. (the owls will die when there are less food/or no food)...

5)




| when $x$ | $x^2$ | $y$ |
|---|---|---|
| 1 | 1 | 4 |
| 2 | 4 | 11 |
| 3 | 9 | 22 |
| 4 | 16 | 35 |
| 5 | 25 | 56 |
| 6 | 36 | 80 |
| 7 | 49 | 107 |
| 8 | 64 | 140 |
| 9 | 81 | 175 |
| 10 | 100 | 215 |

Yes the data provided supports the proportionality $y \propto x^2$.