103lab2solns W 19

Calculus I (Wilfrid Laurier University)

Would you like this assignment to be marked? **YES** **NO**

MA103 Lab Report 2 - Limits Continued

Name:_______________________ Student Number:__________________ Winter 2019

1. [6 *marks*] Consider the function $f(x) = \dfrac{x-3}{\sqrt{x+9x^2}}$.

   (a) State the domain of f using interval notation.

   $x + 9x^2 > 0 \Rightarrow x(1+9x) > 0 \Rightarrow x < -\dfrac{1}{9}$ or $x > 0$

   $\therefore dom_f = \left(-\infty, -\dfrac{1}{9}\right) \cup (0, \infty)$

   (b) Determine any horizontal asymptotes of f.

$$\begin{aligned}\lim_{x\to\infty} f(x) &= \lim_{x\to\infty} \frac{x-3}{\sqrt{x+9x^2}} \\ &= \lim_{x\to\infty} \frac{x-3}{\sqrt{x+9x^2}} \div \frac{x}{\sqrt{x^2}} \\ &= \lim_{x\to\infty} \frac{1-\frac{3}{x}}{\sqrt{\frac{1}{x}+9}} \\ &= \frac{1}{\sqrt{9}} = \frac{1}{3}\end{aligned}$$

$$\begin{aligned}\lim_{x\to-\infty} f(x) &= \lim_{x\to-\infty} \frac{x-3}{\sqrt{x+9x^2}} \\ &= \lim_{x\to-\infty} \frac{x-3}{\sqrt{x+9x^2}} \div \frac{x}{-\sqrt{x^2}} \\ &= \lim_{x\to-\infty} \frac{1-\frac{3}{x}}{-\sqrt{\frac{1}{x}+9}} \\ &= \frac{1}{-\sqrt{9}} = -\frac{1}{3}\end{aligned}$$

   $\therefore$There are horizontal asymptotes along $y = -\dfrac{1}{3}$ and $y = \dfrac{1}{3}$.

2. [5 *marks*] Using Maple, plot the function $f(x) = e^x - 5x - 8$ on the interval $[-5, 5]$:

   f(x):=??? (substitute appropriately for **???**)

   plot(f(x),x=−5..5)

   (a) Consider the equation $e^x = 5x + 8$. Based on your Maple results, are there any solutions to this equation? Explain your reasoning.

   Yes. The two x-intercepts of $f(x)$ are the solutions to the equation $e^x = 5x + 8$.

   (b) Use Maple to calculate $f(2)$ and $f(4)$. Round your answers to three decimal places.

   f(2); evalf(%); [Repeat for $f(4)$.] $f(2) =$ -10.611 $f(4) =$ 26.598

   (c) Explain whether the Intermediate Value Theorem be used to verify that there is at least one root to the equation $e^x = 5x + 8$ in the interval $(2, 4)$. If it can, state its conclusion.

   Since f is continuous on the closed interval $[2, 4]$ and $f(2) < 0 < f(4)$, the IVT applies.

   Thus, by the IVT, there must exist at least one $c \in (2, 4)$ such that $f(c) = 0$. Hence, there is a root to the equation $e^x = 5x + 8$ in the interval $(2, 4)$.

   (d) Use Maple to determine the value of $c \in (2, 4)$ such that $f(c) = 0$. Round your answer to 3 decimal places.

   fsolve(f(c)=0,c=2..4) $c =$ 3.172

3. [3 *marks*] The velocity (in feet per second) of a skydiver t seconds after jumping is given by

$$v(t) = -\sqrt{\frac{32}{k}}\left(\frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}\right) \qquad \text{where } k \text{ is a constant.}$$

(a) Determine the **limiting velocity** of the skydiver, $\lim_{t\to\infty} v(t)$.

$$\begin{aligned}\lim_{t\to\infty} v(t) &= \lim_{t\to\infty} -\sqrt{\frac{32}{k}}\left(\frac{1-e^{-2t\sqrt{32k}}}{1+e^{-2t\sqrt{32k}}}\right)\\ &= -\sqrt{\frac{32}{k}}\left(\frac{1-0}{1+0}\right) = -\sqrt{\frac{32}{k}} \text{ ft/sec}\end{aligned}$$

(b) Evaluate the limiting velocity when the skydiver is in (i) a head first position ($k = 0.00016$) and (ii) a spread eagle position ($k = 0.001$). Round your answers to the nearest whole number and include units.

The limiting velocity is:

(i) $-\sqrt{\dfrac{32}{0.00016}} \doteq -447$ ft/sec when a skydiver is head first and

(ii) $-\sqrt{\dfrac{32}{0.001}} \doteq -179$ ft/sec when a skydiver is spread eagle.

4. [4 *marks*] Find the derivative of $f(x) = \dfrac{1}{1-x}$ using the limit definition.

$$\begin{aligned}f'(x) &= \lim_{h\to0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to0}\frac{1}{h}\left(\frac{1}{1-(x+h)}-\frac{1}{1-x}\right)\\ &= \lim_{h\to0}\frac{1}{h}\left(\frac{1}{1-x-h}-\frac{1}{1-x}\right)\\ &= \lim_{h\to0}\frac{1}{h}\left(\frac{1-x-(1-x-h)}{(1-x-h)(1-x)}\right) = \lim_{h\to0}\frac{1}{h}\left(\frac{1-x-1+x+h}{(1-x-h)(1-x)}\right)\\ &= \lim_{h\to0}\frac{1}{h}\left(\frac{h}{(1-x-h)(1-x)}\right)\\ &= \lim_{h\to0}\frac{1}{(1-x-h)(1-x)} = \frac{1}{(1-x)^2}\end{aligned}$$

5. [3 *marks*] Suppose that the sales revenue for a new video game depends on the selling price $p \geq 0$ (in dollars) of the game and is given by $R(p) = \dfrac{10000p}{5625+p^2}$ (in millions of dollars).

(a) Define $R(p)$ in Maple and use it to determine $R(25)$, $R(225)$, $R'(25)$, and $R'(225)$. Include units in your answer and round results to 2 decimal places where necessary.

R(p):=???

$R(25) = \$40$ million	**R(25)**	$R(225) = \$40$ million
$R'(25) = \$1.28$ million/dollar	**R'(25)**	$R'(225) = -\$0.14$ million/dollar

(b) Interpret the results in part (a). [Hint: The marginal revenue $R'(p_0)$ approximates the change in revenue R when the selling price is increased by \$1 from p_0 to $p_0 + 1$.]

Although the revenues are equal when the selling prices are $p = \$25$ and $p = \$225$, the revenue will increase by approximately \$1.28 million if the price is increased from \$25 to \$26 while it will decrease by approximately \$140 000 if the price is increased from $p = \$225$ to \$226.

Grade: $\dfrac{21}{21}$