Practice Final Test

MATH1104, summer 2018

Carleton University, Ottawa
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Mocking Test

- **First name and last name:**. . . . .
- **Student no:** . . . .

Note:

- The exam has 9 questions. Answer each question on its own page. The last four pages of the pamphlet are for rough work. **Do not use your own scrap papers during this exam.** If you write any solution on the last 4 pages, indicate clearly under the question.
- **Calculators are not allowed**. The exam does not involve any heavy calculation. Do not use your cell phone, text books, lecture notes, and any other material.
- Do not forget to write your name and student number on this page.
- Do not separate pages of this pamphlet from each other.
- Points of each question is written in front of it. Note that some questions may have a couple of parts. Do not forget to answer to them all.

Good Luck!

Question 1. B1. [10 marks] Given a system of linear equations as follows:

$$\begin{cases} x_1 & -2x_2 & -x_3 & +3x_4 & = & 1 \\ 2x_1 & -4x_2 & +x_3 & & = & 5 \\ x_1 & -2x_2 & +2x_3 & -3x_4 & = & 4 \end{cases}$$

(a) Write the above system into matrix equation $Ax = b$. (2 points)

(b) Find the reduced row echelon form of the augumented matrix of the above system. (3 points)

(c) Using (b) to solve the above system. Express your solutions in parametric vector form. (2 points)

Use this page to answer **Question 1**.

Question 2. Let $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

(a) Find the inverse of A. (3 points)

(b) Let $a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, a_2 = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}, a_3 = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$. Does a_1 belong to the span of $\{a_2, a_3\}$? (explain your justification) (1 points)

(c) Is $\lambda = 0$ an eignevalue of A? (explain your justification) (1 points)

Use this page to answer **Question 2**.

Question 3. Let $A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$.

(a) Diagonalize A. (4 points)

(b) Use your answer to (a), to find A^2. (2 points)

Use this page to answer **Question 3**.

Question 4. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation which interchanges $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and maps $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find the affiliated matrix of T. (3 points)

Question 5. Let $A = \begin{bmatrix} 1 & -2 & 2 & 1 & -3 \\ -3 & 1 & -2 & -4 & 0 \\ 4 & -1 & 4 & 7 & -1 \end{bmatrix}$ whose reduced echelon form is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

(a) Find a basis for col(A). (3 points)

(b) Find a basis for nul(A). (3 points)

(c) Verify that the dimension of the column space of A plus the dimension of the null space of A is equal to the number of columns of A. (1 points)

Use this page to answer **Question 5**.

Question 6. Let $A = \begin{bmatrix} -5 & 0 & 2 \\ 1 & -2 & 3 \\ 6 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 101 & 202 & 303 \\ 404 & 505 & 606 \\ 707 & 808 & 909 \end{bmatrix}$.

(a) Find $\det(A)$. (3 points)

(b) Find $\det(A^{101}B^{100})$. (2 points)

Question 7. Let $u = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -2 \end{bmatrix}$.

(a) Find a vector w of length 1 which is in the same direction of v.(2 points)

(b) Write u as the sum of two orthogonal vectors, one in direction of w and one orthogonal to w. (4 points)

Question 8. Suppose that $\{v_1, v_2, \ldots, v_{10}\}$ are then linearly independent vectors in $\mathbb{R}^{10}$. Let V be the subspace of $\mathbb{R}^{10}$ which is spanned by $\{v_1, v_2, \ldots, v_{10}\}$.

(a) Let A be the 10×10 matrix $[v_1 v_2 \ldots v_{10}]$. Is A invertible? (2 points)

(b) Is V equal to $\mathbb{R}^{10}$ or not? Why? (2 points)

Question 9. Let $A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$.

(a) Find the characteristic polynomial of A. (1 points)

(b) Find all (complex) eigenvalues of A. (1 points)

(c) Find a basis for each eigenspace of A. (4 points)

Use this page to answer **Question 9**.

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