Conic Sections on Polar Coordinates

takes the form $r = f(\theta) = \dfrac{\square}{1 + \Delta \cdot \text{trig}(\theta)}$, $\square, \Delta$ are #'s

trig θ will be $\cos\theta$ or $\sin\theta$

This form is when the Focus $F = (0,0)$

Let $p = \dfrac{\square}{\Delta}$, eccentricity ("e") $= |\Delta| \geq 0$,

If trig $\theta = \sin\theta$	vs trig $\theta = \cos\theta$
then directrix is a horizontal line	then directrix is a vertical line
$y = \dfrac{\square}{\Delta}$	$x = \dfrac{\square}{\Delta}$

Note: directrix is "based at" point p.

eccentricity tells us	conic Type
$e < 1$	ellipse
$e = 1$	parabola
$e > 1$	hyperbola

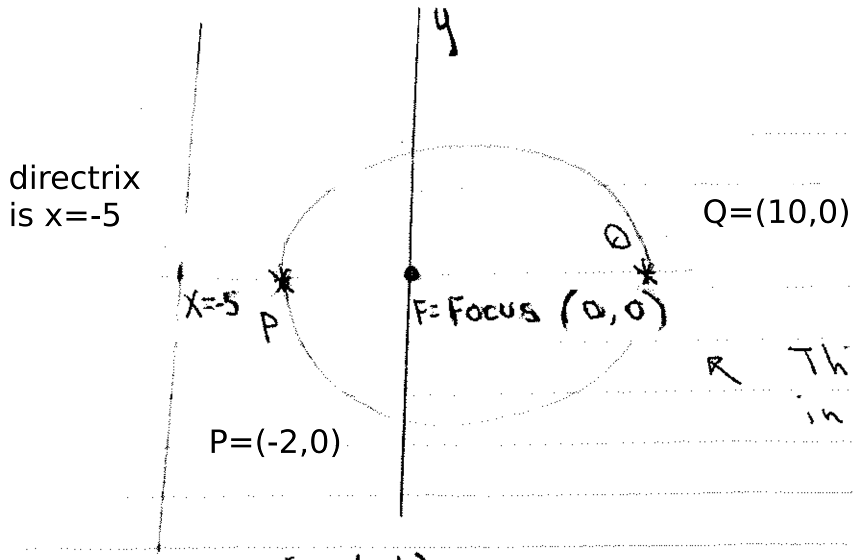
e.g. $r = \dfrac{10}{3 - 2\cos\theta} \Rightarrow r = \dfrac{\boxed{\frac{10}{3}}}{1 + \boxed{\frac{-2}{3}}\cos\theta}$

vertical directrix because of cos

$\dfrac{\square}{\Delta} = -5$ d: $\left| \dfrac{\frac{10}{3}}{-\frac{2}{3}} \right| = 5$, vertical directrix $\rightarrow x = -5$

i.e. $e = |\Delta|$, $\Delta = -\frac{2}{3}$.
$= 2/3 < 1$

So it is an ellipse.



This picture is in the xy-plane

Conic Sections (cont'd)

review

$$r = \frac{\square}{1 + \Delta \text{"trig"}\,\theta} \quad ; \quad e = |\Delta| \geq 0$$

directrix is "based at" a point, $p = \dfrac{\square}{\Delta}$

if "trig θ" is $\cos\theta \Rightarrow$ x=p

" " " $\sin\theta \Rightarrow$ y=p

$e < 1$ ellipse (In fact, e=0 is a circle)

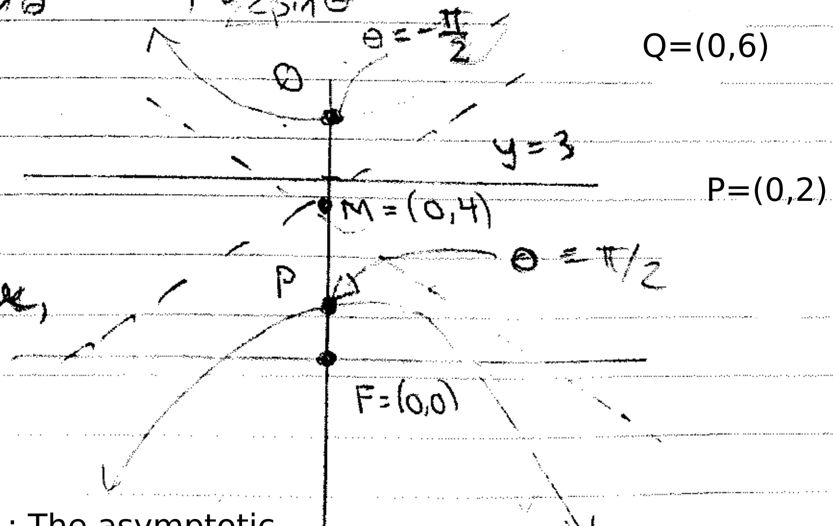
$e = 1$ parabola

$e > 1$ hyperbola

example: $r = \dfrac{12}{2 + 4\sin\theta} = \dfrac{6}{1 + 2\sin\theta} \Rightarrow e = 2$

directrix

at $y = + \dfrac{\boxed{6}}{2} = 3$



To find direction of "asymptotic" lines,

determine when $r \to \infty$

$1 + 2\sin\theta = 0$

$2\sin\theta = -1$

$\sin\theta = -1/2$

$\theta = -\pi/6, \; 7\pi/6$

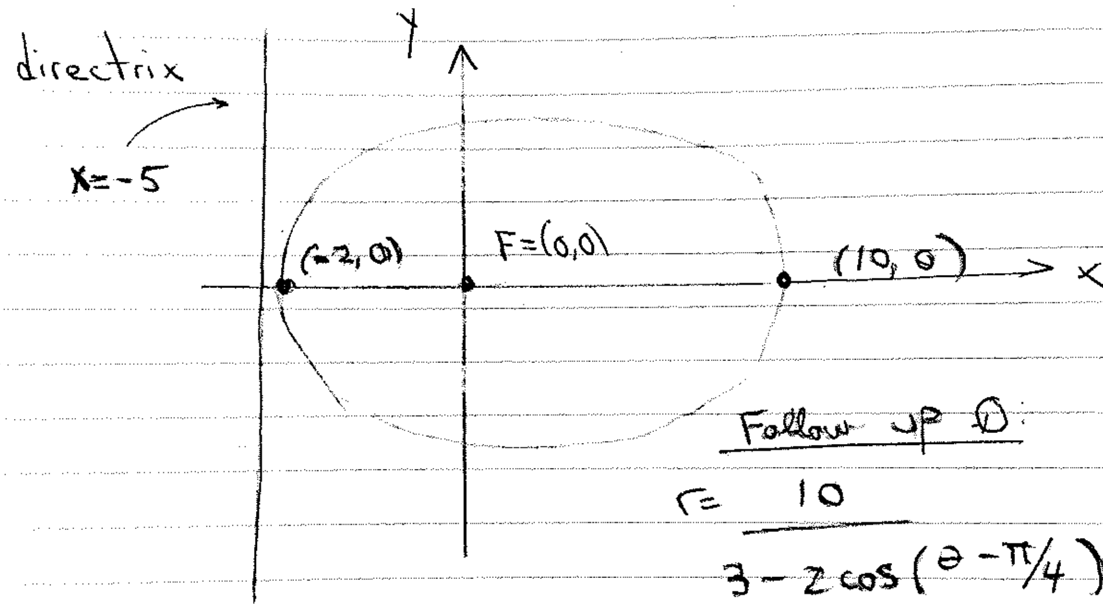
Note #1: The asymptotic lines will be parallel to the rays given by these two angles

Note#2: M is halfway between P and Q.
(M is intersection pt of asymptotic lines)

example : $r = \dfrac{10}{3 - 2\cos\phi} = \dfrac{10/3}{1 - \frac{2}{3}\cos\phi}$

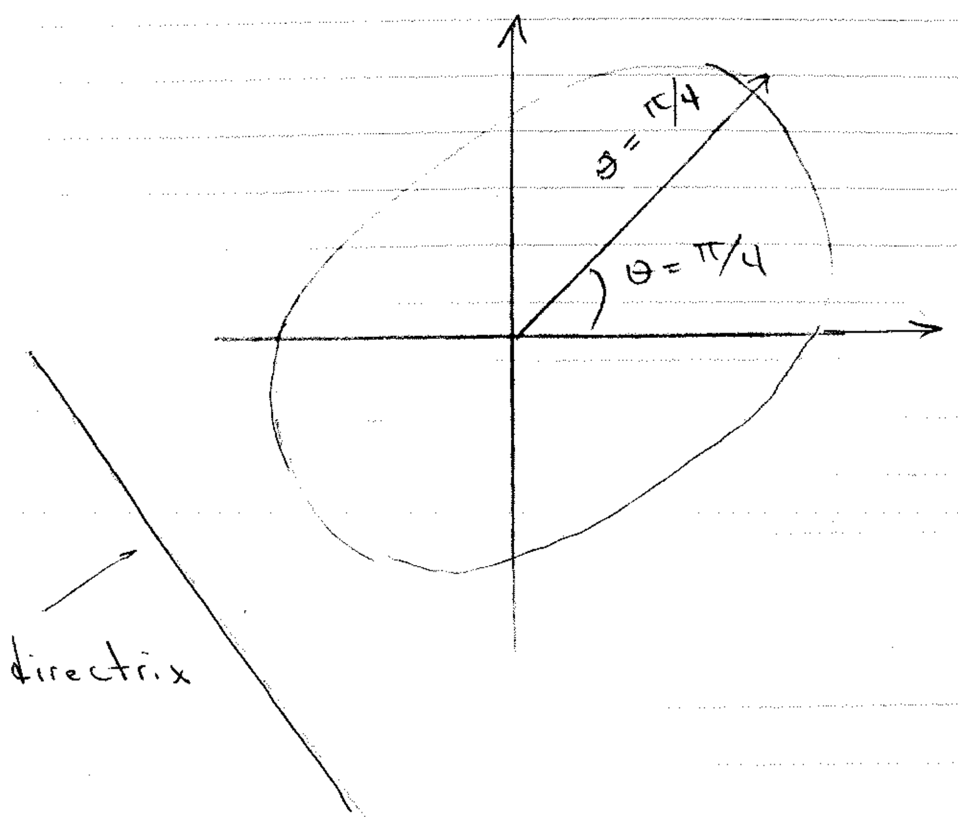
$e = \left|-\frac{2}{3}\right| = \frac{2}{3}$, $p \Rightarrow x = \dfrac{\frac{10}{3}}{-\frac{2}{3}} = -5$

$e = \frac{2}{3} < 1$, ellipse



Follow up θ:

$r = \dfrac{10}{3 - 2\cos(\theta - \pi/4)}$



Let $\phi = \theta - \frac{\pi}{4}$

This is a rotation by $\frac{\pi}{4}$ of the prev. curve, in a counter-clockwise direction

Areas with Parametric Curves

(area in x, y co-ordinates), $dA = y\,dx$

Area=height * width =h * w but only if both h, w are positive

or $dA = x\,dy$

say $x = f(t)$
$y = g(t)$

$$A = \int y\,dx \Rightarrow A = \int g(t)\,\frac{dx}{dt}\,.\,dt$$

$$= \int_{t=t_0}^{t=t_1} |g(t)\,f'(t)|\,dt$$

Note: * assume $\xrightarrow[t_0 \quad t_1]{}$ to ensure that dt>0

* and make sure heights and widths positive by using abs value.

example: $x = t^5$, $y = t^{15}$. Find the area bounded by the curve, the x-axis and between $-32 \le x \le 1$

let $f(t) = t^5$
$g(t) = t^{15}$

$$A = \int |t^{15}\,5t^4|\,dt = 5\int_{-2}^{1} |t^{19}|\,dt$$

$$= -5\int_{-2}^{0} t^{19}\,dt + 5\int_{0}^{1} t^{19}\,dt = 5\int_{0}^{-2} t^{19}\,dt + 5\int_{0}^{1} t^{19}\,dt$$

$$= 5\left[\frac{(-2)^{20}}{20} + \frac{1}{20}\right]$$