University of Ottawa - Department of Physics

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PHY2361 - Winter 2020

Assignment 1

/20

SOLUTION

Exercise 1: (8 pts)

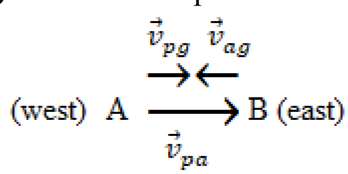
An airline, all of whose planes fly with an airspeed of 300 km/h, serves three cities, A, B, and C, where B is 500 km due east of A, and C is 500 km due north of A. On a certain day there is a steady wind of 200 km/h from the east.

(a) Find the time needed for a round trip from A to B and back.

(b) Find the time needed for a round trip from A to C and back.

Solution:

a) - On A $\rightarrow$ B part :



$\vec{v}_{pa}$ = velocity plane trough air

$\vec{v}_{pg}$= velocity plane trough ground

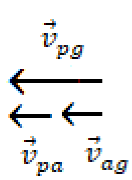
$\vec{v}_{ag}$= velocity air trough ground

we have,

$\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$

$v_{pg} = v_{pa} - v_{ag} \Rightarrow t_{A\rightarrow B} = \frac{\mathrm{d}}{v_{pg}} = \frac{\mathrm{d}}{v_{pa} - v_{ag}}$

- On B $\rightarrow$ A part :

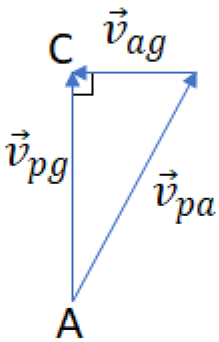


$\Rightarrow \vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$

$v_{pg} = v_{pa} + v_{ag} \Rightarrow t_{B\to A} = \frac{\mathrm{d}}{v_{pg}} = \frac{\mathrm{d}}{v_{pa}+v_{ag}}$

$$\Rightarrow t_{A\to B\to A} = \frac{\mathrm{d}}{v_{pa} - v_{ag}} + \frac{\mathrm{d}}{v_{pa} + v_{ag}} = \frac{500\ \mathrm{km}}{(300 - 200)\ km/h} + \frac{500\ \mathrm{km}}{(300 + 200)\ km/h}$$
$$= 5h + 1h = \mathbf{6}\ \boldsymbol{h}$$

b) We have :



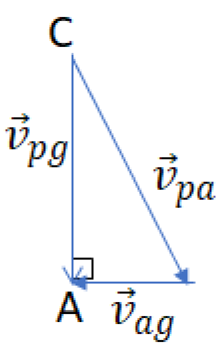
$\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$

$v_{pg} = \sqrt{v_{pa}^2 - v_{ag}^2}$

And,

$t_{A\to C} = \frac{\mathrm{d}}{v_{pg}}$

On C → A (way back) :



v_{pg} on C→A is same as on A → C.

$$\Rightarrow t_{A\to C\to A} = 2t_{A\to C} = \frac{2\mathrm{d}}{v_{pg}} = \frac{2\mathrm{d}}{\sqrt{v_{pa}^2 - v_{ag}^2}} = \frac{2(500\ \mathrm{km})}{\sqrt{300^2 - 200^2(\frac{km}{h})^2}} = \mathbf{4.47}\ \boldsymbol{h}$$

Exercise 2: (6 pts)
Imagine a train that moves at 0.6 *c*. A man on the ground outside (in frame S at rest) sees man *A*, at the rear of the carriage, start shooting at *B*, who is standing about 10 m ahead of him. After 12.5 ns, he sees *B* start shooting back.
But the passengers all claim that *B* shot first. Explain why.
(Find the difference in time between the events, as seen in S' moving in the usual configuration at 0.6 *c* along the x-axis of S. Which event occurs first in S'?)
Solution:

Event 1 = Man A start shooting B

Event 2 = Man B start shooting A

From Lorentz transformations, event 1 has coordinates in S' frame (see Figure below):

$x_1' = \gamma(x_1 - vt_1)$ (1)

$t_1' = \gamma(t_1 - vx_1/c^2)$ (2)

Event 2 has coordinates in S' frame:

$x_2' = \gamma(x_2 - vt_2)$ (3)

$t_2' = \gamma(t_2 - vx_2/c^2)$ (4)

(4)-(2) => $t_2' - t_1' = \gamma\left(t_2 - \frac{vx_2}{c^2}\right) - \gamma(t_1 - vx_1/c^2)$

=> $t_2' - t_1' = \gamma\left(t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1\right)$

=> $t_2' - t_1' = \gamma\left(t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1\right)$

=> $t_2' - t_1' = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}\left(t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1\right)$

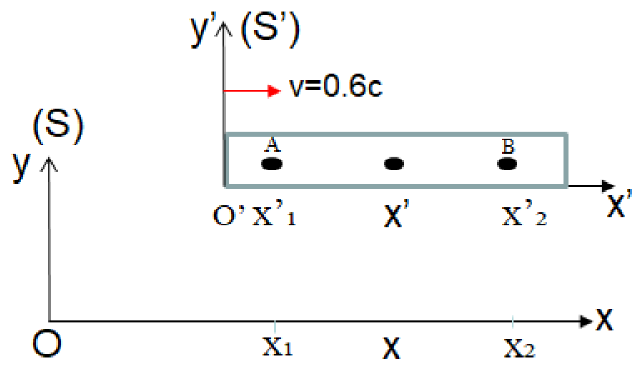
=> $t_2' - t_1' = \frac{1}{\sqrt{1-(\frac{0.6c}{c})^2}}\left(12.5ns - \frac{0.6c}{c^2}(10m\right)$

=> $t_2' - t_1' = \frac{1}{\sqrt{1-(0.6)^2}}\left(12.5ns - \frac{0.6}{c}(10m\right)$

=> $t_2' - t_1' = \frac{1}{\sqrt{1-(0.6)^2}}\left(12.5ns - \frac{0.6}{3\times10^8}(10m\right)$

=> $\boldsymbol{t_2' - t_1' = -9.375\ ns < 0}$

So, Event 2 occurs first in S'frame (*B* shot first).



Exercise 3: (6 pts)
Textbook Ch. 2 Ex. 34
In the frame in which they are at rest, the number of muons at time t is given by
$N = N_0 e^{-t/\tau}$
where No is the number at t = 0 and τ is the mean life-time 2.2 μs.
(a) If muons are produced at a height of 4.0 km, heading toward the ground at 0.93c, what fraction will survive to reach the ground?
(b) What fraction would reach the ground if classical mechanics were valid?

Solution :

a) In rest frame of muons, height of mountain is contracted,
$l = \gamma^{-1} l_0$ where l_0 = proper length (height) = 4.0 km, and mountain approaches muons with speed v = 0.93 c, so takes time
$t = l/v = \gamma^{-1} l_0/v = \gamma^{-1} l_0/0.93c$ to pass.
With,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.93^2}} \cong 2.71$$

$$t = \frac{\gamma^{-1} l_0}{0.93c} = \frac{4.10^3 m}{(2.72)(0.93)(3.10^8)} = 5.27 \times 10^{-6} s$$

=> $N/N_0 = e^{-t/\tau} = e^{-\left(\frac{5.27.10^{-6}}{2.2.10^{-6}}\right)} = \mathbf{9.1 \times 10^{-2} = 9.1\%}$

b) If one assumes, classically, that time and length measurements are independent of reference frame, then could calculate

$N(t)/N_0 = e^{-t'/\tau}$
where,

$$t' = \frac{l_0}{v} = \frac{l_0}{0.93c} = \frac{4.10^3 m}{0.93(3.10^8) m/s} = 1.43.10^{-5} s$$

and τ = 2.2 μs.

=> $N(t)/N_0 = e^{-t'/\tau} = e^{-\left(\frac{1.43.10^{-5} s}{2.2.10^{-6} s}\right)} = \mathbf{1.5 \times 10^{-3} = 0.15\ \%}$