Practice Final Exam Winter 2020

PHY2361

Problem 1:

Relativity:

Galaxy A is approximately 2 million light years (ly) distant from Earth as measured in the Earth-linked frame. In the following consider the distance from Earth to Galaxy A to be exactly 2.10^6 ly and neglect any relative motion of the Earth and A.
a) A one-way trip of a rocket ship to Galaxy A takes 2.01×10^6 years (as measured in the Earth-linked frame) to cover the distance of 2.00×10^6 ly. How long does the trip last as measured in the rocket ship frame?
b) What is the rocket ship speed on Trip 1 as measured in the Earth-linked frame? Express this speed as a fraction of the speed of light, ($\beta = v/c$).
c) The rocket ship one-way trip to Galaxy A takes 2.001×10^6 y, How long does this trip last as measured in rocket frame? What is the speed of the rocket ship in this case (expressed as before)?

Problem 2:

Relativity:

What is the speed v of an electron whose relativistic kinetic energy equals its rest energy?

Problem 3:
Photoelectric effect:

Light of wavelength 450 nm is incident on a metallic surface. If the stopping potential for the photoelectric effect is 0.40 V, find
a) the maximum energy of the emitted electrons,
b) the work function.

Problem 4:
Compton scattering:

A scattered photon with energy of 120 keV undergoes Compton scattering with an electron. The recoiling electron has an energy of 40 keV. Find
a) the wavelength of the incident photon,
b) the angle at which the photon is scattered,
c) the recoil angle of the electron.

The Compton wavelength of the electron is:

$$\lambda_e = \frac{h}{m_e c} = 2.43\times10^{-12}\text{ m}$$

The mass of the electron is:

$$m_e = 511\text{ keV}/c^2$$

Problem 5:
Wave function:

In a region of space, a particle with mass m and with zero energy has a time-independent wave function

$$\psi(x) = Axe^{-x^2/L^2}$$

where A and L are constants.
Determine the potential energy U(x) of the particle.

Problem 6:
Infinite square well:

A particle is in the nth energy state $\Psi_n(x)$ of an infinite square well potential with width L.

a) Determine the probability $P_n(1/a)$ that the particle is confined to the first 1/a of the width of the well.
b) Comment on the n-dependence of $P_n(1/a)$.

Problem 7:
Quantum system:

An electron is described by the wave function

$$\psi(x) = \begin{cases} 0 & \text{for } x < 0 \\ Ce^{-x}(1 - e^{-x}) & \text{for } x > 0 \end{cases}$$

where x is in nm and C is a constant.

a) Determine the value of C that normalizes Ψ(x).
b) Where is the electron most likely to be found? That is, for what value of x is the probability of finding the electron the largest?

Problem 8:
Harmonic Oscillators:

A particle of mass m in a harmonic oscillator potential

$$V(x) = \frac{m\omega_0^2}{2}x^2$$

has an initial wave function

$$\psi(x,0) = \frac{1}{\sqrt{2}}\left[\phi_0(x) + i\phi_1(x)\right]$$

where φ_0 and φ_1 are the n=0 and n=1 normalized eigenstates for the harmonic oscillator. Write down $\psi(x, t)$ and $|\psi(x, t)|^2$. (For this part, you may leave the expression in terms of φ_0 and φ_1.)

Problem 9:
Eigenfunctions of $\hat{L}^2$ and $\hat{L}_z$

Working in spherical coordinates, $\hat{L}^2$ *and* $\hat{L}_z$ take the form,

$$\hat{L}^2 = -\hbar^2 \quad \frac{1}{\sin\theta}\frac{\partial}{\partial\theta} \quad \sin\theta\frac{\partial}{\partial\theta} \quad + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$$

and the first few spherical harmonics, Y_{lm}, take the form,

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}}$$

$$Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta$$

Show that these functions are eigenfunctions of both $\hat{L}^2$ *and* $\hat{L}_z$, and compute the corresponding eigenvalues

Problem 10:
Problem 62 ch 8 (Harris)

Identify the different total angular momentum states (j, m_j) allowed a 3d electron in a hydrogen atom.