

1. Identify and fix all hazards present in $f = (ab + \bar{a}c)(c + \bar{b}) + ab$. Recheck if the expression you have obtained is indeed hazard-free. 8 Marks

no hazards for C ; need to check for a, b.
 Choose c first to restrict

$$(ab + \bar{a}c)(c + \bar{b}) + ab$$

Can reuse this level

$\begin{array}{l} c=0 / \\ ab\bar{b} + ab \\ b=0 / \quad \backslash b=1 \\ 0 \quad \quad a \end{array}$	$\begin{array}{l} \backslash c=1 \\ (ab + \bar{a}) + ab \\ b=0 / \quad \backslash b=1 \\ \bar{a} \quad \quad a + \bar{a} \\ \text{static 1 hazard} \\ \text{for } b=c=1 \end{array}$
$\begin{array}{l} ab\bar{b} + ab \\ a=0 / \quad \backslash a=1 \\ 0 \quad \quad b\bar{b} + b \\ \text{dynamic hazard} \\ \text{for } a=1 \\ c=0 \end{array}$	$\begin{array}{l} (ab + \bar{a}) + ab \\ a=0 / \quad \backslash a=1 \\ 1 \quad \quad b \end{array}$

Static 1 hazard for $b=c=1$; can be marked by ORing with bc
 dynamic hazard for $a=1; c=0 \rightarrow$ need to mark the static hazard $b\bar{b}$ in $(ab + \bar{a}c)(c + \bar{b})$ for $a=1$ and $c=0$ and fix it with $(\bar{a} + c)$

Expression without hazards.

$$(ab + \bar{a}c)(c + \bar{b})(\bar{a} + c) + ab + bc$$

*continued

Recheck to make sure no new hazards were introduced.

$$(ab + \bar{a}c)(c + \bar{b})(\bar{a} + c) + ab + bc$$

$$c=0 / \quad \backslash c=1$$

Reuse \leftarrow $ab\bar{b}\bar{a} + ab$ $(ab + \bar{a})b + ab + b$

$$b=0 / \quad \backslash b=1 \quad b=0 / \quad \backslash b=1$$
$$0 \quad a \quad \bar{a} \quad 1$$

$$ab\bar{b}\bar{a} + ab$$

$$(ab + \bar{a})b + ab + b$$

$$a=0 / \quad \backslash a=1$$

$$a=0 / \quad \backslash a=1$$

$$0 \quad b \quad b \quad b$$

No hazards !!

Final answer

$$(ab + \bar{a}c)(c + \bar{b})(\bar{a} + c) + ab + bc$$

2. Find the reduced state table for the following FSM.

7 Marks

State	Next State		Output
	x = 0	x = 1	
A	B	F	0
B	A	B	0
C	G	E	0
D	H	F	1
E	F	C	0
F	E	D	1
G	C	G	1
H	D	H	0

Two three way merges are possible.

C - E - H
and D - F - G

State table Next State

State	x=0	x=1	output
A	B	DFG	0
B	A	B	0
CEH	DFG	CEH	0
DFG	CEH	DFG	1

3. Express $z = (\bar{a} + \bar{b} + c)(a + \bar{c} + d)(b + a + \bar{d})(\bar{d} + \bar{a} + c + \bar{b})$ in sum-of-products form.

6 Marks

$\bar{z} = ab\bar{c} + \bar{a}c\bar{d} + \bar{b}\bar{a}d + da\bar{c}b$

ab \ cd	00	01	11	10
00		1	1	1
01				1
11	1	1		
10				

Invert

ab \ cd	00	01	11	10
00	1			
01	1	1	1	
11			1	1
10	1	1	1	1

Kmap for \bar{z}

$z = a\bar{b} + ac + \bar{a}\bar{c}\bar{d} + \bar{a}bd$

5. Loop the following K-maps using heuristics discussed in class to minimize the number of gates and gate inputs. Write down the resulting boolean expressions for each function indicating the shared terms. 9 Marks

$f(a, b, c)$

	b		a	
	0	d	1	0
c	1	0	1	1

$g(a, b, c)$

	b		a	
	0	1	d	1
c	d	1	1	0

$h(a, b, c)$

	b		a	
	0	1	1	1
c	1	d	0	1

$f = ab + J$
 $g = b + K$
 $h = b\bar{c} + J + K$

$J = c\bar{b}$
 $K = a\bar{c}$

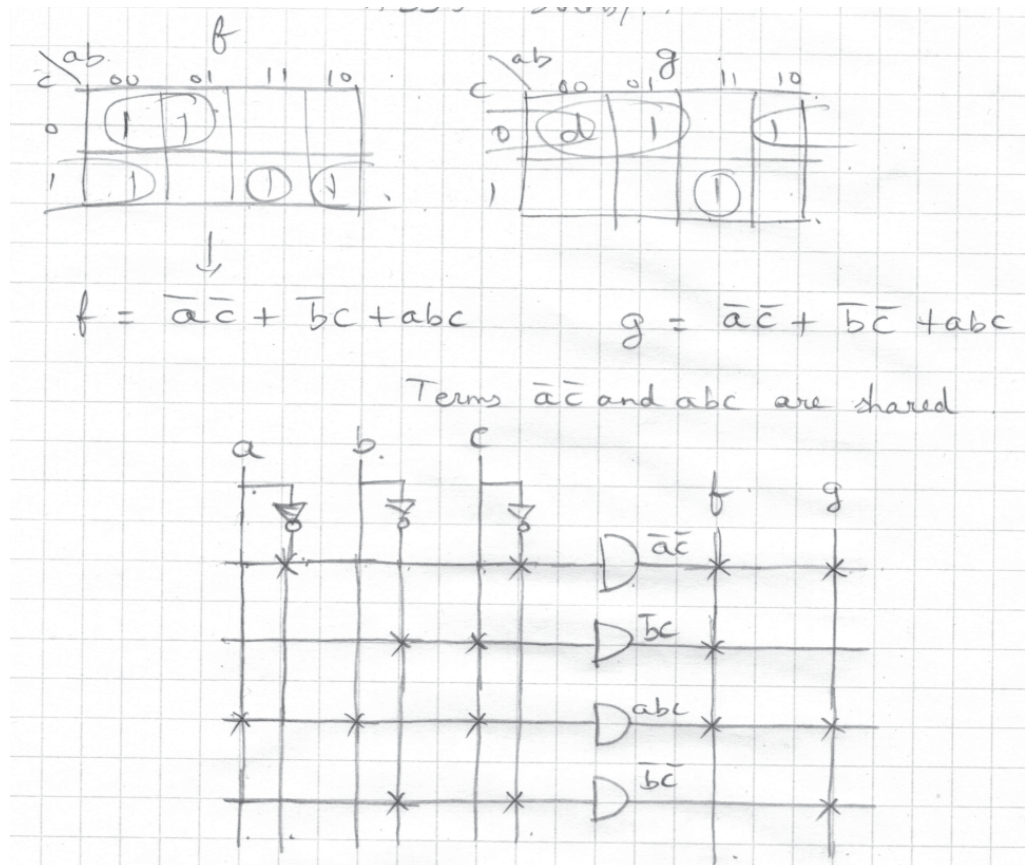
6. Implement the following two functions on the same PLA making sure to use a minimum number of rows in the implementation. Show your implementation through a symbolic diagram of the PLA. 6 Marks

$f(a, b, c)$

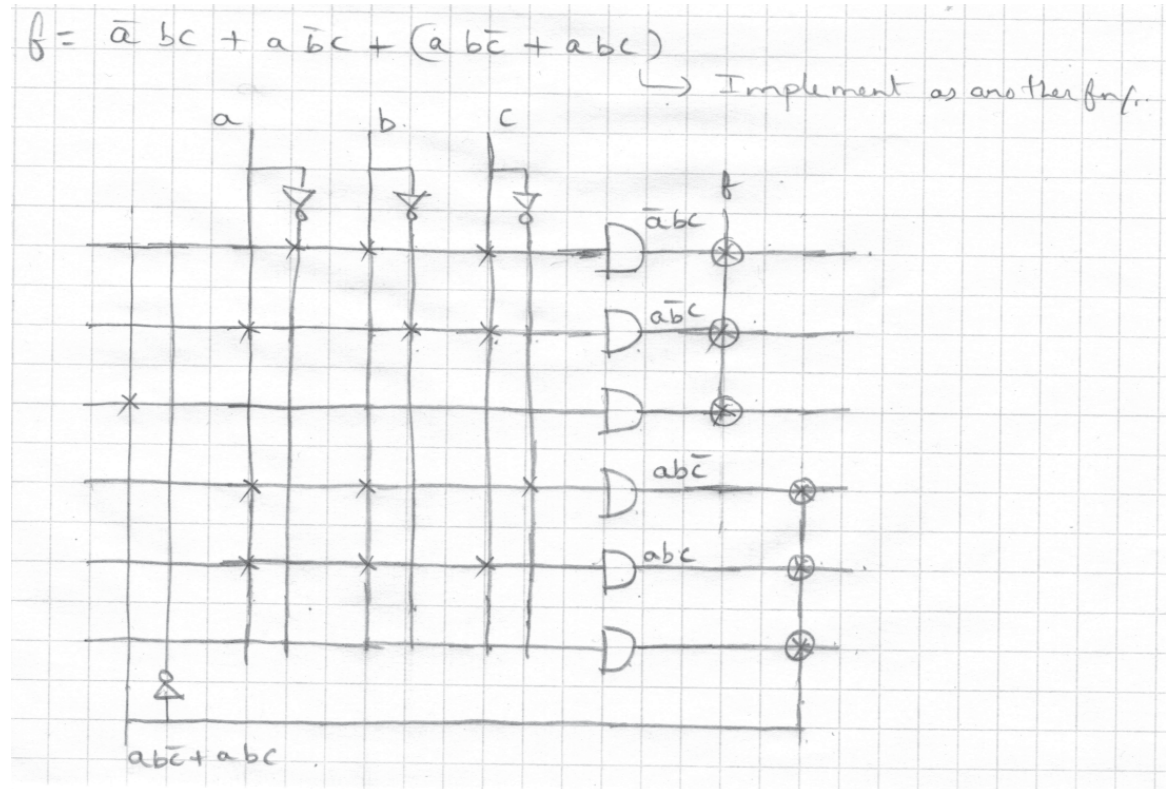
	b		a	
	00	01	11	10
c	0	1	0	1
1	1	0	1	1

$g(a, b, c)$

	b		a	
	00	01	11	10
c	0	1	0	1
d	1	0	1	0



7. Implement $f = \bar{a}bc + a\bar{b}c + (abc\bar{c} + abc)$ using a PAL. Assume that for this PAL three terms are 'or'ed together in the OR-plane for each function. Show your implementation through a symbolic diagram of the PAL. 5 Marks



8. Implement the function $\bar{a}bc + \bar{b}(ad + \bar{a}\bar{d}) + ab\bar{c}d$ using only 2-input MUXs and inverters. Use parameters a, b, c etc. as control signals for the 1st (i.e. top), 2nd, 3rd etc. levels of MUXs respectively. 5 Marks

