Experiment B: Stresses in a Thin-Walled Pressure Vessel

February 26th, 2020

MAAE 2202 B

Purpose

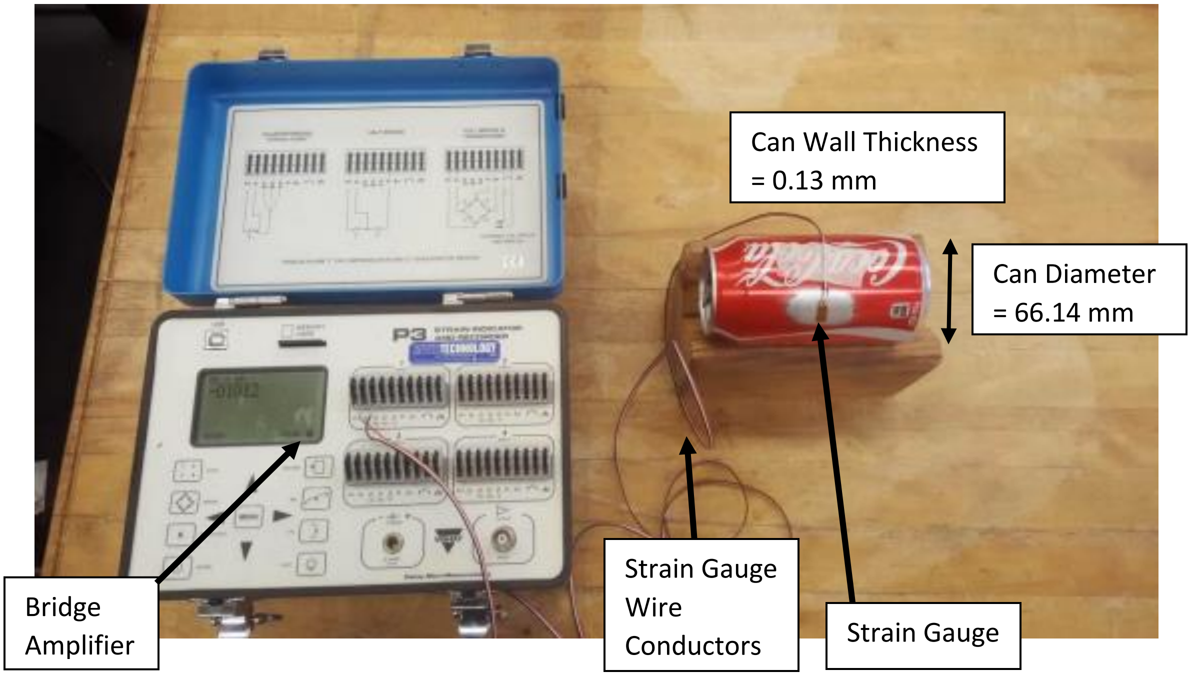
The purpose of this lab is to use given strain values obtained through strain gage application to analyze the relationship between the hoop and longitudinal stress for an aluminum can and in turn using the stress values to calculate the can's internal pressure. In doing this, conclusions can be drawn about the hoop to longitudinal stress ratio, the safety factor for an aluminum can, the safety features the can possesses to prevent over-pressurization, and how the calculated values compare to established theoretical values.

Procedure

Refer to "MAAE 2202 Mechanics of Solids I Laboratory Experiments" pages 14-15 for procedure

Apparatus

Figure 1 illustrates the experiment test set up. Hoop and Longitudinal Strain gages are glued to the pop can, while three conductor strain gage wires connect the two strain gages to the bridge amplifier.



Equipment Used

1. Soldering iron
2. Aluminum plate (2” × 2”)
3. PC board
4. Tweezers
5. Wire strippers
6. Pop can
7. Strain gage
8. Safety Goggles
9. Hot Glue Gun
10. Bridge amplifier
11. Scissors
12. Micrometer
13. Consumables: Solder, 3M Scotch Brite pad, 3M Scotch Tape (“crystal” not “invisible”), degreaser, paper towl swabs, wire, cyanoacrylate adhesive, adhesive activator

Theory and Analysis

The theory of this experiment focuses on finding the internal pressure of a thin walled soda can based on the surface strain. When thin walled cylinders feel an internal pressure, there is stress developed in two directions, called hoop and longitudinal stresses respectively. When considering the equilibrium of a cross sectional area of the thin walled cylinder that is under pressure, the hoop and longitudinal pressures are assumed to be constant through the wall of the cross-sectional area.

The force equilibrium for the longitudinal stress is found by simply multiplying the longitudinal area by the internal pressure, or by taking the longitudinal stress and wall thickness and multiplying by the circumference

$$(1)\quad F = P * \pi * d^2/4 \ \textbf{ or } \ F = \sigma_{long} * \pi * d * t_{wall}$$

The force equilibrium for the hoop stress can be found by taking the hoop area by the internal pressure, or by taking the hoop stress and wall thickness and multiplying by 2 times the length.

(2) $F = P * L * d$ **or** $F = 2 * \sigma_{hoop} * \pi * d * t_{wall}$

The hoop stress can be found by applying equilibrium to a unit length of the cylindrical area under pressure, using the wall internal pressure P, diameter d, and wall thickness t

(3) $\sigma_{hoop} = Pd/2t_{wall}$

The longitudinal stress can be found by applying equilibrium to a circular cross-sectional area

(4) $\sigma_{long} \pi d t_{wall} = P \pi d^2/4$ **or** $\sigma_{long} = Pd/4t$

Additionally, using Aluminum's known elastic modulus of 69 GPa [1] and Equation (5)

(5) $E = \sigma/\varepsilon$

The two stresses can be calculated using the known strain data. Using these equations, along with the known physical properties of the area under pressure, the internal pressure can be calculated.

Sample Calculations

The full rough draft of calculations can be seen in Appendix A

A. Determine the hoop stress, longitudinal stress, and internal pressure of the soda can using the strain data, wall thickness, and modulus of elasticity.

Table 1: Given Data

Aluminum Modulus of Elasticity	69 GPa
Hoop strain	-257 με
Longitudinal strain	-51 με
Can diameter	66.14 mm
Average wall thickness	0.13 mm

1. Hoop Stress

$\sigma_{hoop} = E\, \varepsilon_{hoop}$

$\sigma_{hoop} = (69 * 10^9 \text{ Pa}) * (-257*10^{-6})$

$\sigma_{hoop} = -1.77 * 10^7 \text{ Pa}$

2. Longitudinal Stress

$\sigma_{long} = E\ \varepsilon_{long}$

$\sigma_{long} = (69 * 10^9\ Pa) * (-51 * 10^{-6})$

$\sigma_{long} = -3.52 * 10^6\ Pa$

3. Internal Hoop Pressure

$\sigma_{hoop} = Pd/2t_{wall}$

$\varepsilon_{hoop} = Pd/2t_{wall}\ E$

$-257 * 10^{-6} = P*(0.06614m)/(2*0.00013m*(69*10^9\ Pa))$

$P = -69709.4\ Pa$

4. Internal Longitudinal Pressure

$\sigma_{long} = Pd/4t_{wall}$

$\varepsilon_{long} = Pd/4t_{wall}\ E$

$\varepsilon_{long} = P*(0.06614m)/(4*0.00013m*(69*10^9\ Pa))$

$-51 * 10^{-6} = P*(0.06614m)/(4*0.00013m*(69*10^9\ Pa))$

$P = -27666.8\ Pa$

B. Determine a Safety Factor for Aluminum using the Ultimate Allowable Tensile Strength

Ultimate Tensile Strength for Aluminum = 310 MPa [2]

Safety Factor = Ultimate Tensile Strength/Actual Stress

Safety Factor = $310 * 10^6$ Pa/ $(1.77*10^7)$

Safety Factor = 17.5:1

C. Ratio between Hoop Stress and Longitudinal Stress

Ratio = Hoop Stress/Longitudinal Stress

$= 1.77*10^7/3.52*10^6$

= 5:1

Table 2: Calculated Results

Hoop Stress	$-1.77*10^7$ Pa
Longitudinal Stress	$-3.52*10^6$ Pa
Internal Hoop Pressure	-69709.4 Pa
Internal Longitudinal Pressure	-27666.8 Pa
Safety Factor	17.5
Ratio Between Hoop and Longitudinal Stress	5:1

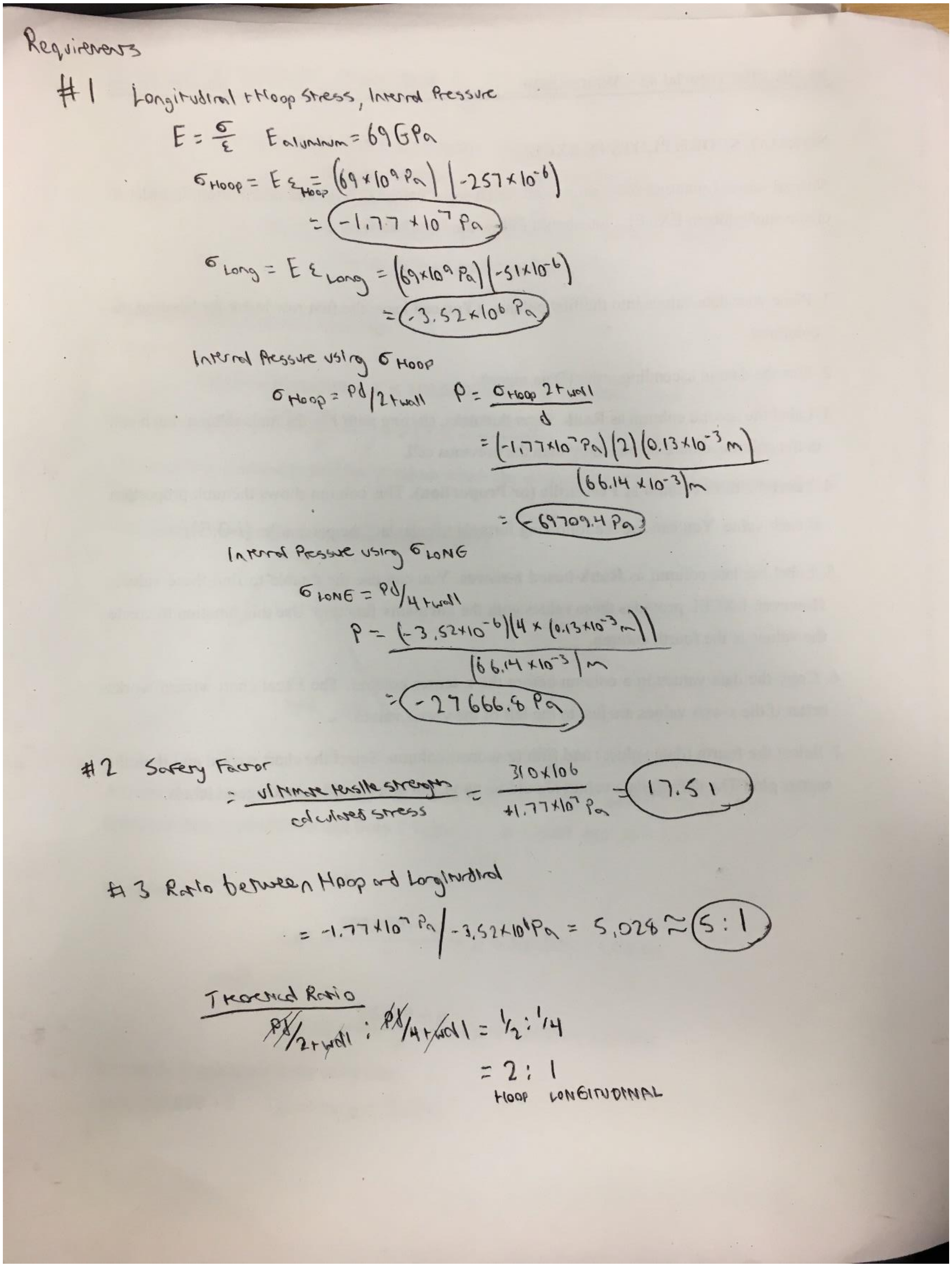
Discussion

From the results shown in Table 2, there are a few key takeaways from this lab. To start, all calculated stresses and pressures are negative because the strain value given on the bridge amplifier is negative. The strain values are negative because as the can is opened, the internal pressure decreases as liquid is released from the can, and thus there is also less stress and strain on the can. The ratio between hoop stress and longitudinal stress was 5:1, because the hoop strain from the bridge amplifier was five times the longitudinal strain, when theoretically the stress ratio should be 2:1, proven by comparing equations (3) and (4) where the longitudinal pressure is divided by 4 times the wall thickness while the hoop pressure is divided by only 2 times the wall thickness (calculation shown in Appendix A). There are many possible sources of error that account for this discrepancy, including poor conduction from the wires between the strain gauge and the amplifier, small errors in measurement of the can diameter and wall thickness, fluctuating readings on the bridge amplifier, and poor application of the strain gauge to the can. Additionally, the safety factor of 17.5:1 is very high, considering engineering safety factors are usually in the 1.5 to 2 range. Because the calculation of the safety factor considers the calculated stress, all possible sources of error for the first calculations are also applicable. Aluminum cans like the one tested possess safety features that guard against over pressurization, in order to keep the can from exploding. They have a concave dome shape at the bottom, which allows them to withstand more pressure than if the bottom were flat or concave outwards ie. takes a higher pressure to deform the can. Additionally, the can has a cylindrical shape because this avoids the corners and edges associated with a rectangle or square. These corners and edges are the weakest points in any object, and the cylindrical nature of the can mitigates the weak links [2].

Conclusion

This lab analyzed the hoop and longitudinal stresses, as well as the internal pressure of an aluminum can, using given hoop and longitudinal strains found using strain gage application techniques. The calculated hoop stress of $-1.77*10^7$ Pa was approximately 5 times the longitudinal stress of $-3.52*10^6$ Pa, which made sense given the hoop strain was also 5 times greater. This 5:1 ratio was much higher than the theoretical ratio of 2:1, and there are multiple sources of error that can account for this as mentioned in the discussion section. The can's internal pressure was also calculated, and the structure of the can was examined to show how the internal pressure can be so high with such minimal wall thickness. The cylindrical shape of the can, as well as the concave domed bottom, are both safety features that allow the can to have such a high internal pressure without deforming.

Appendix A: Rough Calculations for Longitudinal and Hoop Stress, Internal Pressure, Safety Factor, and Hoop to Longitudinal Stress Ratio



Requirements

#1 Longitudinal + Hoop Stress, Internal Pressure

$E = \frac{\sigma}{\varepsilon}$ $E_{aluminum} = 69\,GPa$

$\sigma_{Hoop} = E\varepsilon_{Hoop} = (69\times10^{9}\,Pa)(-257\times10^{-6})$

$= \boxed{-1.77\times10^{7}\,Pa}$

$\sigma_{Long} = E\varepsilon_{Long} = (69\times10^{9}\,Pa)(-51\times10^{-6})$

$= \boxed{-3.52\times10^{6}\,Pa}$

Internal Pressure using σ_{Hoop}

$\sigma_{Hoop} = pd/2t_{wall}$ $p = \frac{\sigma_{Hoop}\,2t_{wall}}{d}$

$= \frac{(-1.77\times10^{7}\,Pa)(2)(0.13\times10^{-3}\,m)}{(66.14\times10^{-3})m}$

$= \boxed{-69709.4\,Pa}$

Internal Pressure using σ_{LONG}

$\sigma_{LONG} = pd/4t_{wall}$

$p = \frac{(-3.52\times10^{-6})(4\times(0.13\times10^{-3}\,m))}{(66.14\times10^{-3})m}$

$= \boxed{-27666.8\,Pa}$

#2 Safety Factor

$= \frac{\text{ultimate tensile strength}}{\text{calculated stress}} = \frac{310\times106}{+1.77\times10^{7}\,Pa} = \boxed{17.51}$

#3 Ratio between Hoop and Longitudinal

$= -1.77\times10^{7}\,Pa / -3.52\times10^{6}\,Pa = 5.028 \approx \boxed{5:1}$

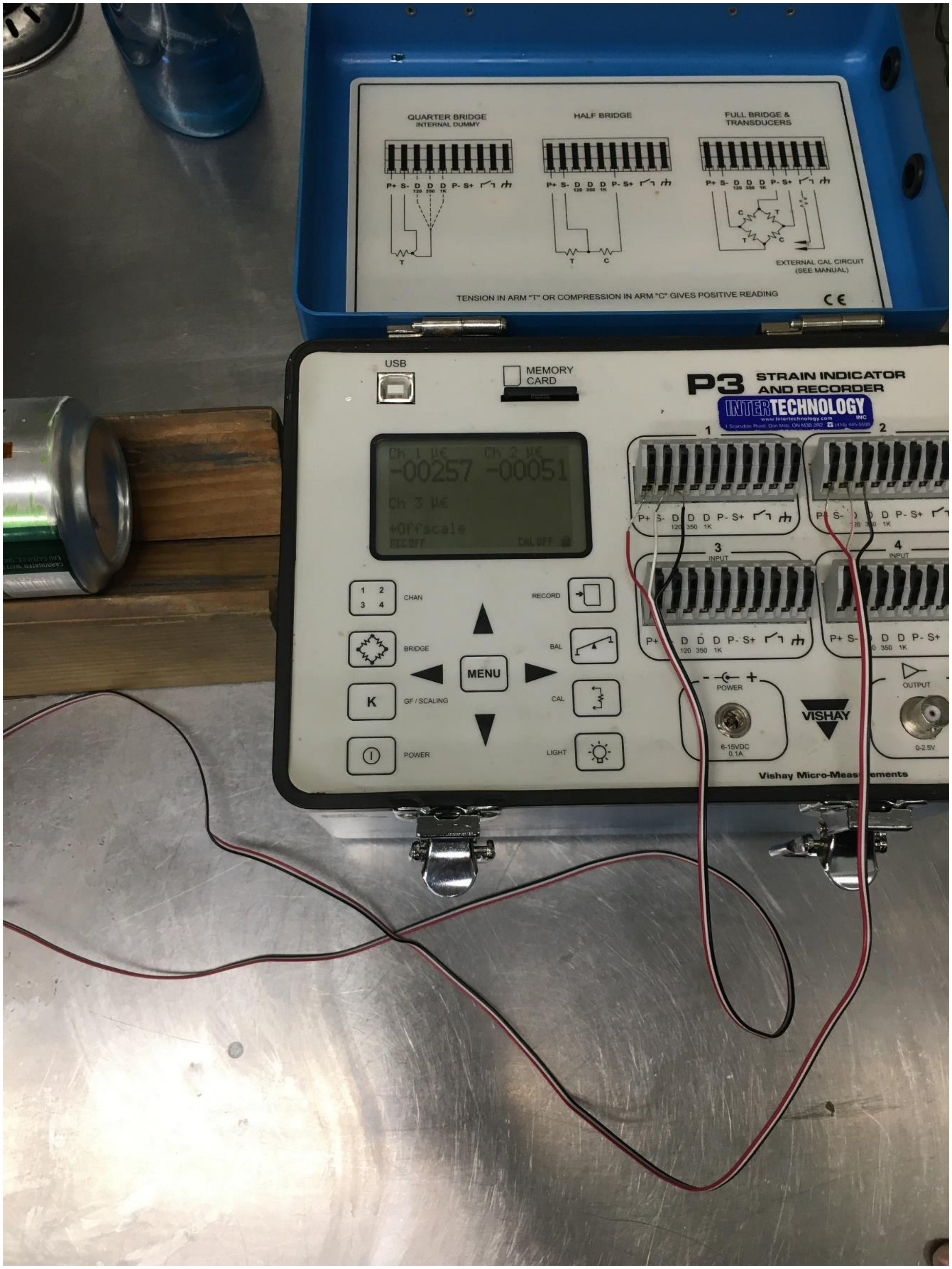
Theoretical Ratio

$\cancel{pd}/2\cancel{t_{wall}} : \cancel{pd}/4\cancel{t_{wall}} = 1/2 : 1/4$

$= 2 : 1$

HOOP LONGITUDINAL

Appendix B: Bridge Amplifier Data



References

[1] "Young's Modulus - Tensile and Yield Strength for common Materials", *Engineeringtoolbox.com*, 2020. [Online]. Available: https://www.engineeringtoolbox.com/young-modulus-d_417.html. [Accessed: 03- Mar- 2020].

[2] “ASM Material Data Sheet", *Asm.matweb.com*, 2020. [Online]. Available: http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA6061T6. [Accessed: 04- Mar- 2020].

[3] Ward-Bailey, "The surprising science behind the aluminum soda can", *The Christian Science Monitor*, 2020. [Online]. Available: https://www.csmonitor.com/Science/Science-Notebook/2015/0414/The-surprising-science-behind-the-aluminum-soda-can. [Accessed: 02- Mar- 2020].

[4] “MAAE 2202 Mechanics of Solids I” pages 11-19, 2020. [Online]. [Accessed: 03-Mar-2020]