MATH 1007 B, TEST 1
Time: 50 Minutes, Total Mark: 25
SOLUTION

Name: ..... Student #: .....

Question 1. [4] Find the *domain* and the *range* of each one of the following functions.

(a) [2] $f(x) = \dfrac{x+3}{x^2-9}$

Solution: $f(x) = \frac{1}{x-3}$. So, $Domain(f) = (-\infty, 3) \cup (3, \infty)$

$Range(f) = (-\infty, 0) \cup (0, \infty)$

(b) [2] $g(x) = x^4 + 10$

Solution: $Domain(f) = (-\infty, \infty)$

$Range(f) = [10, \infty)$

Question 2. [5] Which ones of the following functions are Odd and which ones are Even? Also identify which one is symmetric about the $y-$axix and which one is symmetric about the origin $(0, 0)$.

(a) [3] $f(x) = \dfrac{\sin x}{x^3}$

Solution: Remember that $\sin x$ is an odd function.
$f(-x) = \dfrac{\sin(-x)}{(-x)^3} = \dfrac{\sin x}{x^3}$. So, f is an even function and it is symmetric about y-axis.

(b) [2] $g(x) = x^{-3}$

Solution: Observe that $g(-x) = (-x)^3 = -x^3 = -g(x)$. So, g is odd and as a result it is symmetric about the origin.

Question 3. [8]

(a) [4] If $\cos x = \frac{-\sqrt{2}}{2}$ on $[0, \pi]$ then what is $\sin x$? What is $\tan x$?
Hint: Remember that $\sin^2 x + \cos^2 x = 1$ and that $\tan x = \frac{\sin x}{\cos x}$.
Solution: $\sin^2 x = 1 - \frac{1}{2} = \frac{1}{2}$. So, $\sin x = +\frac{\sqrt{2}}{2}$ or $\sin x = \frac{-\sqrt{2}}{2}$. But since $x \in [0, \pi]$, so $\sin x = +\frac{\sqrt{2}}{2}$. As a result $\tan x = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$.

(b) [4] Find all values of x in the interval $[0, \pi]$ such that

$$\tan^3(x + \pi) + 1 = 0$$

Hint: Remember that $\tan x$ is periodic and also use the result of Part (a) of this question.

Solution: The period of $\tan x$ is π. So, $\tan^3(x + \pi) = \tan^3 x$. So, we have $\tan x = -1$. Till here the solution is complete. However, we can obtain x as follows
$x = \pi - \frac{\pi}{4}$.

Question 4. [8] By using the exponent rules simplify the the below terms as much as possible. (Write some of the steps of your calculations)

(a) [2] $\dfrac{(9)^{-3}}{(9)^{18}}.3^3$

Solution: $\dfrac{(9)^{-3}}{(9)^{18}}.3^3 = 3^{-42}.3^3 = 3^{-39}$

(b) [2] $\dfrac{(x-1)\sqrt{x-1}}{\sqrt{(x-1)^3}}$

Solution: $\dfrac{(x-1)\sqrt{x-1}}{\sqrt{(x-1)^3}} = \dfrac{(x-1)^{3/2}}{(x-1)^{3/2}} = 1$

(c) [4] $(x\sqrt{\sin x})^4(x^{-2}\sqrt{1-\cos^2 x})^2$

Hint: Remember that $\sin^2 x + \cos^2 x = 1$.

Solution: $(x\sqrt{\sin x})^4(x^{-2}\sqrt{1-\cos^2 x})^2 = x^4 \sin^2 x\, x^{-4} \sin^2 = \sin^4 x$