

MAT1330 B – Instructor: Elizabeth Maltais

Wednesday, October 3, 2017 : Test #1

Duration: 75 minutes

Family name: _____

First name: _____

Student number : _____

For picking up your graded test:

Circle the DGD you will attend to pick up your test (whether you are registered or not). Tests take at least one week to grade; watch for announcements in class.

# :	DGD1	DGD2	DGD3	DGD4
Day :	Tuesday	Wednesday	Wednesday	Thursday
Start time:	10:00	10:00	1:00	11:30
Room :	MRT 250	SMD 224	STE J0106	FTX 133
TA :	Eric	Rabib	Rabib	Rabib

Please read the following instructions carefully.

- You have 75 minutes to complete this exam.
- This is a closed book exam. Except for Faculty-approved calculators (models: Texas Instruments TI-30* and TI-34*, Casio FX-260* and Casio FX-300*), no notes, cell phones, smartwatches or related devices of any kind are permitted. All such devices, including cell phones, **must be stored in your bag under your desk or at the front of the room for the duration of the exam.**
- Read each question carefully — you will save yourself time and grief later on.
- Questions 1 through 6, and 8, are multiple choice, worth a total of 7 points. **Record your answers to the multiple choice questions in the boxes provided.**
- Questions 7, 9, 10 and 11 are long answer, with number of points as indicated. **You must show your work, your work must be legible and well-justified, and you must record your answers in the spaces provided.**
- Where it is possible to check your work, do so.

• Good luck!

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Marker's use only:

Question	Marks
1-6,8 (/7)	
7 (/2)	
9 (/2)	
10 (/5)	
11 (/4)	
Total (/20)	

1. (1 point) Find all real solutions of $|1 - 2x^2| = 7$.

A. ± 2

C. No solutions exist.

E. $\pm\sqrt{-3}$

B. $\pm\sqrt{2}$

D. ± 3

F. 0

Your answer:

2. (1 point) What is the domain of $f(x) = \frac{\ln(1 - 2x)}{\sqrt{x + 1}}$?

A. all x

C. $-1 < x < \frac{1}{2}$

E. $x > -1$

B. $x > 0$

D. $x \geq \frac{1}{2}$

F. $\frac{1}{2} < x \leq 1$

Your answer:

3. (1 point) Which of the following is equal to $g(x) = \ln(e^x x^3)$ on its domain?

A. $3x \ln(x)$

C. e^3

E. $x + 3 \ln(x)$

B. $3x$

D. $1 + 3 \ln(x)$

F. $\ln(3e)$

Your answer:

4. (1 point) Which of the following is equal to $h(x) = \frac{\sqrt{9x^6 + 4}}{2x^3}$ for $x > 0$?

A. $\sqrt{\frac{9}{4} + x^{-6}}$

B. $\frac{3}{2} + 2x^{-3}$

C. $\sqrt{4.5 + x^{-3}}$

D. $\frac{3}{\sqrt{2}}x^{3/2} + x^{-3/2}$

E. $\sqrt{26x^{12} + 16x^6}$

F. $3x^3 + 2x^{-3}$

Your answer:

5. (1 point) Does there exist a choice of a which makes the following function continuous at $x = 1$? If yes, which one? If not, why not?

$$f(x) = \begin{cases} \ln(2ax) & \text{if } x > 1 \\ \cos(\pi x) & \text{if } x \leq 1 \end{cases}$$

A. yes, $a = e$

B. yes, $a = \frac{1}{2}e^{-1}$

C. yes, $a = \frac{1}{2}$

D. yes, $a = 0$

E. no, $\lim_{x \rightarrow 1} f(x)$ exists, but is not equal to $f(1)$ for any choice of a so f can never be continuous at 1

F. no, $\lim_{x \rightarrow 1} f(x)$ does not exist for any choice of a , so f can never be continuous at 1

Your answer:

6. (1 point) If $h(x) = \frac{1006 + 222x + 317x^2}{4024 + 554x + 634x^2}$ then

A. $\lim_{x \rightarrow \infty} h(x) = \frac{1}{2}$

C. $\lim_{x \rightarrow \infty} h(x) = 1$

E. $\lim_{x \rightarrow \infty} h(x) = -\infty$

B. $\lim_{x \rightarrow \infty} h(x) = \frac{1}{4}$

D. $\lim_{x \rightarrow \infty} h(x) = 0$

F. $\lim_{x \rightarrow \infty} h(x) = \infty$

Your answer:

7. (2 points) Evaluate the following limit, if it exists. You must use algebraic methods and justify your answer mathematically to earn credit for this question.

$$\lim_{x \rightarrow 4^-} \frac{x^2 - x - 12}{|x - 4|}$$

Your answer:

8. (1 point) A study started tracking a herd of elk in northern Alberta. In January 1996, there were 1000 elk in the herd. Over the course of this study, scientists observed that the elk had a natural growth rate of 5% each year. Furthermore, in December each year, an average of 50 elk were killed by hunters. If h_t represents the herd's population (in January), t years after 1996, which of the following DTDS models the dynamics of this herd?

A. $h_{t+1} = 5h_t + 950$

C. $h_{t+1} = 1.05h_t - 50$

E. $h_{t+1} = 0.05h_t - 50$

B. $h_{t+1} = 50h_t - 1000$

D. $h_{t+1} = 1.05h_t + 950$

F. $h_{t+1} = \frac{50}{1000}h_t + 0.05$

Your answer:

9. (2 points) Find all solutions of the inequality. Show your work and justify your reasoning.

$$\frac{x - 3}{x - 5} < \frac{x}{x - 1}$$

Your answer:

10. (1+1+2+1=5 points) A population of snails grows according to a Ricker model, so that the DTDS governing the growth of this population's density is given by

$$x_{t+1} = 2x_t e^{1-0.4x_t}$$

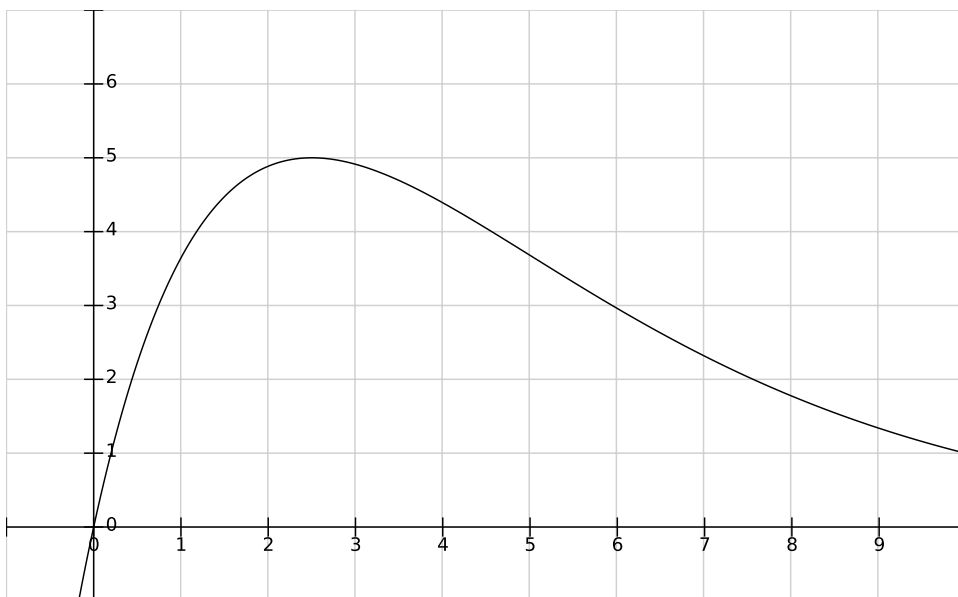
where t is in months and x_t denotes the number of snails per m^2 at time t .

(a) Give the updating function of this DTDS. $f(x) =$

(b) Find all fixed points x^* of this DTDS. Show your work.

Fixed points:

(c) The graph of $y = f(x)$ is drawn below. **Label the axes**, identify the **fixed points** of the DTDS on the graph, then **draw a cobweb** for at least three iterations, starting with an initial population density of $x_0 = 1$. Use your student card as a straight edge.



(d) What do you conclude will happen to this population of snails in the long term? (Your answer should be a complete sentence and should refer to the concepts of stability and fixed points as they pertain to this population of snails and your cobweb.)

11. (1+2+1=4 points) A patient is given a daily dose of the drug ToxicPlacebo, which is absorbed into the patient's system at a constant rate. A DTDS modeling the concentration of drug x_t , in mg/L in the patient's bloodstream each day just after receiving the dose is given by

$$x_{t+1} = 0.9x_t + 0.5,$$

where t is measured in whole days since the treatment began.

(a) Find the fixed point of this DTDS, and give the general solution to this DTDS if the initial value is $x_0 = 1$ mg/L.

Fixed point: $x^* =$

General solution: =

(b) If the initial value is $x_0 = 1$ mg/L, find the number of whole days it takes for the concentration of ToxicPlacebo in the patient's bloodstream to reach at least 4.5 mg/L.

Minimum number of whole days required:

(c) Surprisingly, testing over the year reveals that the concentration of ToxicPlacebo in the patient's bloodstream has stabilized to $x^* = 6$ mg/L. An investigation reveals that the doctor's bad handwriting was to blame, and the administered daily dose d in reality was not actually 0.5 mg/L. What was it?