basic basic basic

a.) $\begin{cases} x_1 = 4x_3 - 4x_5 \\ x_2 = 2x_3 - 2x_5 \\ x_3 = \text{free} \\ x_4 = 2x_5 \\ x_5 = \text{free} \end{cases}$ → vector parametric form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4x_3 - 4x_5 \\ 2x_3 - 2x_5 \\ x_3 \\ 2x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -4 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

Therefore, the basis for Nul(A) is

$$\left\{ \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

b.) Since the 1st, 2nd and 4th columns of A are pivot columns, the basis for Col(A) is:

$$\left\{ \begin{bmatrix} 3 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

c.) Rank A = 5 = $\underbrace{\dim \text{Nul} A}_{3} = 3$

$\begin{vmatrix} 1 & 1+2i \end{vmatrix}$ $= 3-i$

3. False, False, True, True

4. $\begin{vmatrix} 0 & 1 & 0 & 6 \\ -1 & -2 & 5 & -8 \\ 0 & 1 & 0 & -3 \\ 0 & 5 & 1 & -4 \end{vmatrix}$ ① $\longrightarrow \det A = 1$ $\begin{vmatrix} 1 & 0 & 6 \\ 1 & 0 & -3 \\ 5 & 1 & -4 \end{vmatrix}$

c.) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : y = z \right\}$