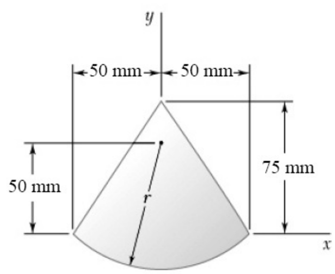
CVG2149 – Civil Engineering Mechanics
Fall 2014

Solutions Tutorial #1

To be solved by summing / subtracting areas

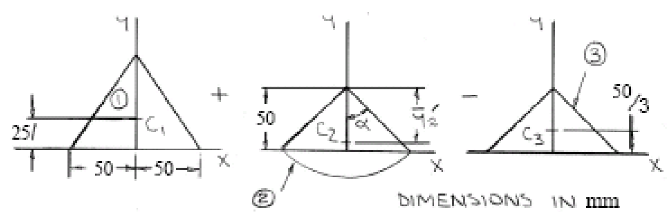


PROBLEM 5.11

Locate the centroid of the plane area shown.

SOLUTION

First note that symmetry implies $\bar{X} = 0$ ◀



$$r_2 = 50\sqrt{2}\ \text{mm},\ \alpha = 45°$$

$$\bar{y}_2' = \frac{2r\sin\alpha}{3\alpha} = \frac{2\left(50\sqrt{2}\right)\sin\left(\frac{\pi}{4}\right)}{3\left(\frac{\pi}{4}\right)} = 42.44\ \text{mm}$$

	A, mm²	$\bar{y}$, mm	$\bar{y}A$, mm³
1	$\frac{1}{2}$ (100) (75) = 3750	25	93,750
2	$\frac{\pi}{4}\left(50\sqrt{2}\right)^2$ = 3926.99	$50 - \bar{y}'$ = 7.56	29688.04
3	$-\frac{1}{2}$ (100) (50) = – 2500	16.67	– 41675
Σ	5176.99		81763.04

Then $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}$ (5176.99 mm²) = 81763.04 mm³ or $\bar{Y}$ = 15.8 mm ◀

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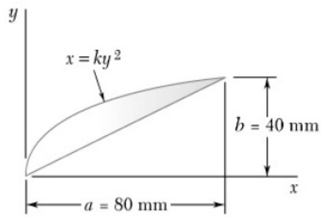
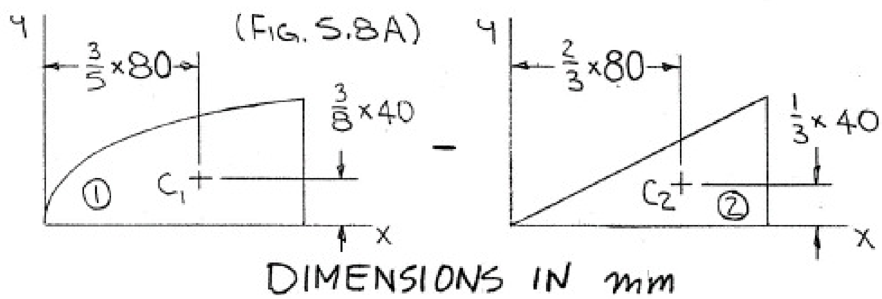


PROBLEM 5.13

Locate the centroid of the plane area shown.

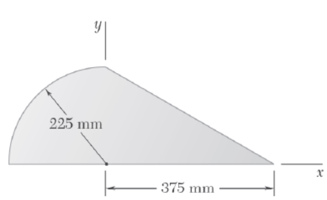
SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{2}{3}(40)(80) = 2133$	48	15	102 400	32 000
2	$-\frac{1}{2}(40)(80) = -1600$	53.33	13.333	–85 330	–21 330
Σ	533.3			17 067	10 667

Then
$$\bar{X}\Sigma A = \Sigma\bar{x}A$$
$$\bar{X}\left(533.3\text{ mm}^2\right) = 17\,067\text{ mm}^3$$
or $\bar{X} = 32.0$ mm ◀

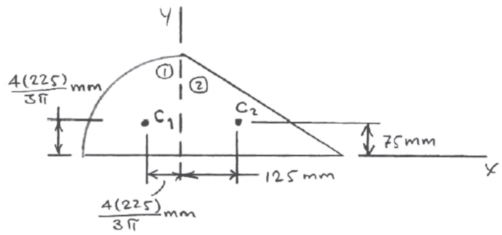
and
$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$
$$\bar{Y}\left(533.3\text{ mm}^2\right) = 10\,667\text{ mm}^3$$
or $\bar{Y} = 20.0$ mm ◀



PROBLEM 5.5

Locate the centroid of the plane area shown.

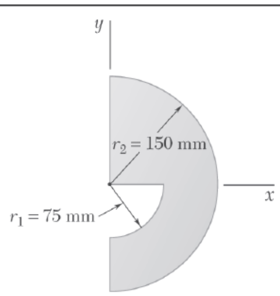
SOLUTION



	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{\pi(225)^2}{4} = 39\ 761$	$-\frac{4(225)}{3\pi} = -95.493$	95.493	−3 796 900	3 796 900
2	$\frac{1}{2}(375)(225) = 42\ 188$	125	75	5 273 500	3 164 100
Σ	81 949			1 476 600	6 961 000

Then $\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1476600}{81949} \text{mm}$ or $\bar{X} = 18.02 \text{ mm}$ ◀

$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{6961000}{81949} \text{mm}$ or $\bar{Y} = 84.9 \text{ mm}$ ◀



PROBLEM 5.24

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

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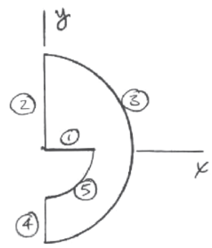
SOLUTION

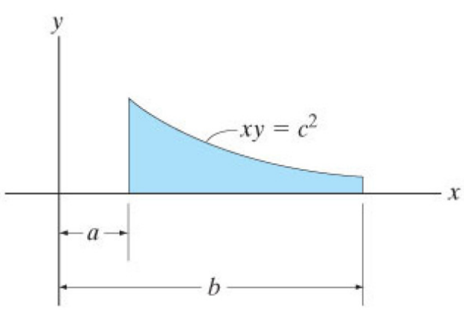


	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	75	37.5	0	2812.5	0
2	150	0	75	0	11 250
3	$(150)\pi = 471.24$	95.492	0	45 000	0
4	75	0	−112.5	0	−8437.5
5	$(75)\frac{\pi}{2} = 117.81$	47.746	−47.746	5625.0	−5625.0
Σ	889.05			53 437	−2812.5

Then $\bar{X} = \frac{\Sigma \bar{x} L}{\Sigma L} = \frac{53\,437}{889.05}$, or $\bar{X} = 60.1$ mm ◀

and $\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{-2812.5}{889.05}$ or $\bar{Y} = -3.16$ mm ◀

To be solved by direct integration



PROBLEM 5.35

Determine by direct integration the centroid of the area shown.

Differential Element: The element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = \frac{c^2}{x}\,dx$$

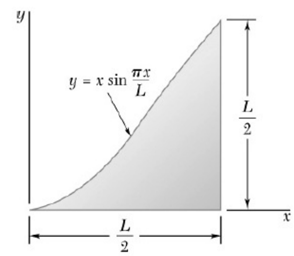
Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = \frac{y}{2} = \frac{c^2}{2x}$.

Area: Integrating,

$$A = \int_A dA = \int_a^b \frac{c^2}{x}\,dx = c^2 \ln x\Big|_a^b = c^2 \ln\frac{b}{a} \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_a^b x\left(\frac{c^2}{x}\,dx\right)}{c^2 \ln\frac{b}{a}} = \frac{\int_a^b c^2\,dx}{c^2 \ln\frac{b}{a}} = \frac{c^2 x\Big|_a^b}{c^2 \ln\frac{b}{a}} = \frac{b-a}{\ln\frac{b}{a}} \qquad \textbf{Ans.}$$

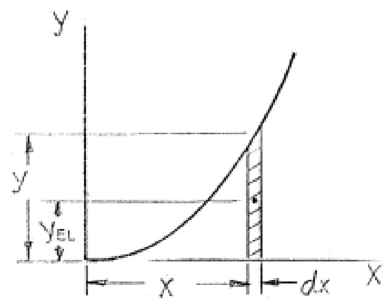
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_a^b \left(\frac{c^2}{2x}\right)\left(\frac{c^2}{x}\,dx\right)}{c^2 \ln\frac{b}{a}} = \frac{\int_a^b \frac{c^4}{2x^2}\,dx}{c^2 \ln\frac{b}{a}} = \frac{-\frac{c^4}{2x}\Big|_a^b}{c^2 \ln\frac{b}{a}} = \frac{c^2(b-a)}{2ab\ln\frac{b}{a}} \qquad \textbf{Ans.}$$



PROBLEM 5.46

Determine by direct integration the centroid of the area shown.

SOLUTION



Have $$\overline{x}_{EL} = x, \quad \overline{y}_{EL} = \frac{1}{2}x\sin\frac{\pi x}{L}$$

and $$dA = y\,dx$$

$$A = \int dA = \int_0^{L/2} x\sin\frac{\pi x}{L}dx = \left[\frac{L^2}{\pi^2}\sin\frac{\pi x}{L} - \frac{L}{\pi}x\cos\frac{\pi x}{L}\right]_0^{L/2} = \frac{L^2}{\pi^2}$$

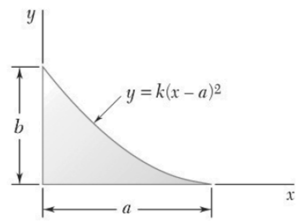
and $$\overline{x} = \int \overline{x}_{EL}dA = \int_0^{L/2} x\left(x\sin\frac{\pi x}{L}dx\right)$$

$$= \left[\frac{2L^2}{\pi^2}x\sin\left(\frac{\pi x}{L}\right) + \frac{2L^3}{\pi^3}\cos\left(\frac{\pi x}{L}\right) - \frac{L}{\pi}x^2\sin\left(\frac{\pi x}{L}\right)\right]_0^{L/2} = \frac{L^3}{\pi^2} - 2\frac{L^3}{\pi^3}$$

Also $$\overline{y} = \int \overline{y}_{EL}dA = \int_0^{L/2}\frac{1}{2}x\sin\frac{\pi x}{L}\left(x\sin\frac{\pi x}{L}dx\right)$$

$$= \frac{1}{2}\left[\frac{2L^2}{\pi^2}x\sin\frac{\pi x}{L} - \left(\frac{L}{\pi}x - \frac{2L^3}{\pi^3}\right)\cos\frac{\pi x}{L}\right]_0^{L/2}$$

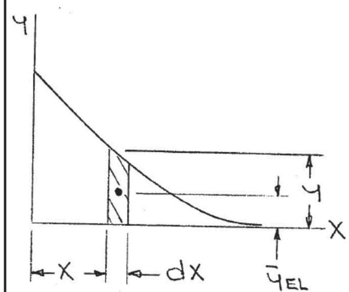
$$= \frac{1}{2}\left[\frac{1}{6}\left(\frac{L^3}{8}\right) - \frac{L^2}{4\pi^2}\left(\frac{L}{2}\right)(-1)\right] = \frac{L^3}{96\pi^2}\left(6 + \pi^2\right)$$



PROBLEM 5.37

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b.

SOLUTION



At $\quad x = 0,\ y = b$

$$b = k(0 - a)^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

Then $\quad y = \frac{b}{a^2}(x - a)^2$

Now $\quad \bar{x}_{EL} = x,\ \bar{y}_{EL} = \frac{y}{2} = \frac{b}{2a^2}(x - a)^2$

and $\quad dA = y\,dx = \frac{b}{a^2}(x - a)^2\,dx$

Then $\quad A = \int dA = \int_0^a \frac{b}{a^2}(x - a)^2\,dx = \frac{b}{3a^2}\left[(x - a)^3\right]_0^a = \frac{1}{3}ab$

and $\quad \int \bar{x}_{EL}\,dA = \int_0^a x\left[\frac{b}{a^2}(x - a)^2\,dx\right] = \frac{b}{a^2}\int_0^a \left(x^3 - 2ax^2 + a^2x\right)dx$

$$= \frac{b}{a^2}\left(\frac{x^4}{4} - \frac{2}{3}ax^3 + \frac{a^2}{2}x^2\right) = \frac{1}{12}a^2b$$

$$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{2a^2}(x - a)^2\left[\frac{b}{a^2}(x - a)^2\,dx\right] = \frac{b^2}{2a^4}\left[\frac{1}{5}(x - a)^5\right]_0^a$$

$$= \frac{1}{10}ab^2$$

Hence $\quad \bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{1}{3}ab\right) = \frac{1}{12}a^2b \qquad \bar{x} = \frac{1}{4}a$ ◀

$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{3}ab\right) = \frac{1}{10}ab^2 \qquad \bar{y} = \frac{3}{10}b$ ◀