[2] **1.** (multiple choice) Which **one** of the following statements is **always** true?

(A) If a function is continuous at a, then it is differentiable at a.

(B) If a function is continuous, then it has an absolute maximum.

(C) If f attains its global maximum on a closed interval at point p, then $f'(p) = 0$.

(D) If a function is differentiable at a, then it is continuous at a.

(E) If $f''(p) = 0$, then f has an inflection point at p.

(F) If p is a critical number of a function f, then f has a local maximum or a local minimum at p.

$f(4) = 4e^{3-4/4} = 4e^2$

[2] **2.** (multiple choice) Consider the function $f(x) = xe^{3-(x/4)}$. Which one of the following is true?

$f'(x) = (1)e^{3-x/4} + xe^{3-x/4}\left(-\frac{1}{4}\right)$

$f'(x) = e^{3-x/4}\left(1 - \frac{x}{4}\right)$

never zero $\quad x = 4$

(A) $(4, 4e^2)$ is a critical point and a local minimum.

(B) $(4, 4e^2)$ is a critical point and a local maximum.

(C) $(4, 4e^2)$ is a critical point and an inflection point.

(D) $(4, 4e^2)$ is not a critical point but it is an inflection point.

4

sign f' $\quad + \quad -$

(E) $(4, 4e^2)$ is a critical point but not a local extremum and not an inflection point.

(F) $(4, 4e^2)$ is not a point on the curve.

[2] **3.** (multiple choice) Suppose $f(x)$ is a continuous function on the interval $[a, b]$. Which **one** of the following statements is **always** true?

(A) $\displaystyle\int_a^b f(x)\,dx = \int_b^a f(x)\,dx.$ $\quad$ No: $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(B) $\displaystyle\int_a^b f(x)\,dx = f(b) - f(a).$ $\quad$ No $\int_a^b f(x)dx = F(b) - F(a)$ where F is an antiderivative of f

(C) $\displaystyle\int_a^b f(x)\,dx = \sum_{i=1}^{n} f(x_i)\Delta x$, where n is a positive integer, $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\Delta x$.

No: $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

(D) If $f(x) \le c$ on $[a, b]$ then $\displaystyle\int_a^b f(x)\,dx \le c(b-a).$

(E) $\int_a^b f(x)\,dx$ is an antiderivative of f. X No: $\int_a^b f(x)dx$ is a number representing net area.

(F) $\int_a^b f(x)\,dx$ is the area of the region bounded by $y=0$, $y=a$, $y=b$ and $y=f(x)$. X

no, it's the net area

[2] **4.** The following table gives its velocity $v(t)$ of a particle, as measured at regular intervals. Estimate the total displacement of the particle from time $t=3$ to $t=6$ s using the trapezoidal rule.

	t_0	t_1	t_2	t_3	t_4	t_5	t_6
t (s)	3	3.5	4.0	4.5	5.0	5.5	6.0
$v(t)$ (m/s)	7	3	1	−4	−1	5	6

$\Delta x = 0.5 \quad n=6$

displacement from $t=3$ to $t=6$ $= \int_3^6 v(t)\,dt$

$\approx T_6 = \frac{\Delta x}{2}\left(v(t_0)+2v(t_1)+2v(t_2)+2v(t_3)+2v(t_4)+2v(t_5)+v(t_6)\right) = \frac{0.5}{2}\left(7+2(3)+2(1)+2(-4)+2(-1)+2(5)+6\right)$

correction

[2] **5.** Find the derivative of $f(x) = \sqrt[3]{\arcsin(e^x)}$.

$f'(x) = \frac{1}{3}\left(\arcsin(e^x)\right)^{-\frac{2}{3}}\left(\frac{1}{\sqrt{1-(e^x)^2}}\right)(e^x)$

[3] **6.** Find the equation of the tangent line to the curve defined implicitly by the equation

$$5e^{xy-1} = 6 - xy^2$$

at the point $(1,1)$.

$(5e^{xy-1})(1\cdot y + x\cdot\frac{dy}{dx} - 0) = 0 - (1)y^2 - x\cdot 2y\cdot\frac{dy}{dx}$

at $(1,1)$: $(5e^{1\cdot1-1})(1+1\frac{dy}{dx}) = -1^2 - 1\cdot2\cdot1\cdot\frac{dy}{dx} \Rightarrow 5(1+\frac{dy}{dx}) = -1-2\frac{dy}{dx} \Rightarrow 5+5\frac{dy}{dx} = -1-2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{6}{7}$ = slope

$\therefore$ equation: $y-y_0 = m(x-x_0)$
$y-1 = -\frac{6}{7}(x-1)$

[3] **7.** If $f(x)$ is a continuous function such that

$$\int_{-2}^{5} f(x)\,dx = 7 \quad\text{and}\quad \int_1^5 f(x)\,dx = 10,$$

$\Rightarrow \int_{-2}^5 f(x)dx = \int_{-2}^1 f(x)dx + \int_1^5 f(x)dx$

$\Rightarrow 7 = \int_{-2}^1 f(x)dx + 10 \Rightarrow \int_{-2}^1 f(x)dx = -3$

what is $\int_{-2}^{1}(2f(x)-4)\,dx$? $= 2\int_{-2}^1 f(x)dx - 4\int_{-2}^1 dx$

$= 2(-3) - 4\left(x\big|_{-2}^{1}\right) = -6-4(1-(-2)) = -6-4(3) = -18$

[3] **8.** Find the linearization $L(x)$ of the function $f(x) = 32x^{-1/2}$ at $x=4$ and use it to give a linear approximation of $\frac{32}{\sqrt{4.5}}$.

at $x=4$ $L(x) = f(4)+f'(4)(x-4)$ $f(4) = \frac{32}{\sqrt{4}} = 16$, $f(x) = 32x^{-1/2} \Rightarrow f'(x) = -16x^{-3/2} \Rightarrow f'(4) = \frac{-16}{(\sqrt{4})^3} = \frac{-16}{8} = -2$

$\therefore L(x) = 16+(-2)(x-4)$ since 4.5 is "near" 4, $f(4.5) = \frac{32}{\sqrt{4.5}} \approx L(4.5) = 16+(-2)(4.5-4) = 16-2(0.5) = 15$.

[4] **9.** Find the following limits.

a) $\lim_{x\to0}(\cos x)^{1/x^2}$ (type 1^∞) $= \lim_{x\to0} e^{\frac{1}{x^2}\ln(\cos(x))} = e^{\left(\lim_{x\to0}\frac{\ln(\cos(x))}{x^2}\right)}$ (type $\frac{0}{0}$) $\overset{\text{Hosp}}{=} e^{\left(\lim_{x\to0}\frac{\frac{-\sin(x)}{\cos(x)}}{2x}\right)} = e^{\left(\lim_{x\to0}\frac{-\tan(x)}{2x}\right)}$ (type $\frac{0}{0}$) $\overset{\text{Hosp}}{=} e^{\left(\lim_{x\to0}\frac{-\sec^2(x)}{2}\right)} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$.

b) $\lim_{x\to0}\frac{e^x-x-1}{\sin(x^2)}$ (type $\frac{0}{0}$) $\overset{\text{Hosp}}{=} \lim_{x\to0}\frac{e^x-1}{\cos(x^2)(2x)}$ (type $\frac{0}{0}$) $\overset{\text{Hosp}}{=} \lim_{x\to0}\frac{e^x}{2\cos(x^2)-2x(2x\sin(x^2))} = \frac{1}{2-0} = \frac{1}{2}$

[2] **10.** Find the derivative of

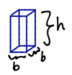
$$A(t) = \int_{\cos(t)}^{1}\sqrt{1+x^3}\,dx.$$

use FTC1 & chain Rule:

$A'(t) = -\frac{d}{dt}\left[\int_1^{\cos(t)}\sqrt{1+x^3}\,dx\right]$

$= -\left(\sqrt{1+(\cos(t))^3}\right)\cdot(-\sin(t))$

[4] **11.** If 1200 cm^2 of material is available to make a box with a square base and an open top, what dimensions will give the largest possible volume of the box?



h= box height

b= base side length

amount material $= b^2 + 4bh = 1200\, cm^2 \Rightarrow h = \frac{1200 - b^2}{4b} = 300b^{-1} - \frac{1}{4}b$

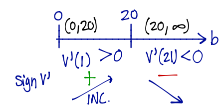
Make a diagram and **label it** carefully.

Volume $= V = b^2 h = b^2(300b^{-1} - \frac{1}{4}b) \Rightarrow V = 300b - \frac{1}{4}b^3$

Justify that your final answer gives a maximum volume.

$V' = 300 - \frac{3}{4}b^2$

$0 = V' \Rightarrow 0 = 300 - \frac{3}{4}b^2 \Rightarrow b^2 = 400 \Rightarrow b = \pm 20$ but $b > 0 \Rightarrow b = 20$



Since V increases for $0 < b < 20$, then decreases for all $b > 20$, We see that V attains its absolute maximum when $b = 20$ cm

$\therefore h = \frac{300}{20} - \frac{20}{4} = 10$ cm

[2] **12.** We want to use Newton's Method to find the root of the polynomial $p(x) = x^5 - 4x + 2$ that is between 1 and 2, starting with initial approximation $x_1 = 1$. Write down the formula for Newton's Method and find x_2 and x_3.

$x_2 = x_1 - \frac{(x_1)^5 - 4x_1 + 2}{5(x_1)^4 - 4}$ $\quad$ $x_3 = x_2 - \frac{(x_2)^5 - 4x_2 + 2}{5(x_2)^4 - 4}$

$p(x) = x^5 - 4x + 2$
$p'(x) = 5x^4 - 4$

$x_{n+1} = x_n - \frac{p(x_n)}{p'(x_n)} = x_n - \frac{(x_n)^5 - 4x_n + 2}{5(x_n)^4 - 4}$

$x_2 = 1 - \frac{1^5 - 4(1) + 2}{5(1^4) - 4} = 2$ $\quad$ $x_3 = 2 - \frac{2^5 - 4(2) + 2}{5(2^4) - 4} = 2 - \frac{26}{76} = \frac{53}{38}$

[15] **13.** Find the following:

$u = \sec(x)$ $\quad du = \sec x \tan x\, dx$

a) $\int \tan^5 x \sec x\, dx$ $= \int \sec x \tan x \cdot (\tan^2 x)^2 dx = \int \sec x \tan x (\sec^2 x - 1)^2 dx = \int (u^2 - 1)^2 du = \int (u^4 - 2u^2 + 1)\, du = \frac{u^5}{5} - \frac{2u^3}{3} + u + C = \frac{\sec^5 x}{5} - \frac{2\sec^3 x}{3} + \sec x + C$

b) $\int t^2 \sin(5t)\, dt$ See next page.

c) $\int_0^1 \frac{2x^2 + x}{(x-2)(x^2+1)}\, dx$ See next page.

d) $\int_{e^2}^{e^4} \frac{2}{x \ln(x)}\, dx$ See next page.

e) $\int \frac{x^3}{\sqrt{1 - x^2}}\, dx =$ See next page.

[4] **14.** Consider a function $y = f(x)$ satisfying the following properties:

- f is a continuous function, except at $x = 4$
- $f(x)$ is not defined at $x = 4$
- $f(0) = 1$
- $\lim_{x \to \infty} f(x) = 2$
- $\lim_{x \to 4^\pm} |f(x)| = \infty$
- $f'(x) = 0$ at $x = 1$ and $x = 3$
- $f'(x) > 0$ for $x < 1$, for $3 < x < 4$ and for $x > 4$
- $f'(x) < 0$ for $1 < x < 3$
- $f''(x) < 0$ for $x < 2$ and $x > 4$
- $f''(x) > 0$ for $2 < x < 4$

on $(-\infty, 1) \cup (3,4) \cup (4, \infty)$

a) State where the function is increasing and decreasing: on $(1,3)$

b) State where it is concave up and concave down.

on $(2,4)$ $\quad$ on $(-\infty, -2) \cup (4, \infty)$

c) State the locations of all local minimums or maximums. local min @ $x=3$ $\quad$ local max @ $x=1$

d) Sketch the graph of f in the space below. Label the special features on the graph, including: asymptotes, inflection points, critical points and local minimums or maximums.

See two pages ahead.

[0] **15.** Just for fun, what is the definition of a "boring" function?

Parts I $u=t^2 \quad v'=\sin(5t)$
$u'=2t \quad v=-\frac{1}{5}\cos(5t)$

13.b) $\int t^2\sin(5t)dt = -\frac{1}{5}t^2\cos(5t) - \int -\frac{2}{5}t\cos(5t)dt$

Parts II $u=t \quad v'=\cos(5t)$
$u'=1 \quad v=\frac{1}{5}\sin(5t)$

$$= -\frac{1}{5}t^2\cos(5t) + \frac{2}{5}\left[\frac{t}{5}\sin(5t) - \int 1\cdot\frac{\sin(5t)}{5}dt\right]$$

$$= -\frac{t^2}{5}\cos(5t) + \frac{2t}{25}\sin(5t) - \frac{2}{25}\left(\frac{-\cos(5t)}{5}\right) + C$$

13.c) $\int_0^1 \frac{2x^2+x}{(x-2)(x^2+1)}dx$

partial fractions $\frac{2x^2+x}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+Bx(x-2)+C(x-2)}{(x-2)(x^2+1)}$

$\Rightarrow 2x^2+x = (A+B)x^2 + (C-2B)x + (A-2C) \Rightarrow A+B=2$
$C-2B=1$
$A-2C=0$

Also, plug in $x=2$: $2(2^2)+2 = A(2^2+1)+0B+0C \Rightarrow 10=5A \Rightarrow A=2$

$\therefore B=2-A=2-2=0$ and $C=1+2B=1+2(0)=1$

$$\Rightarrow \int_0^1 \frac{2x^2+x}{(x-2)(x^2+1)}dx = \int_0^1\left(\frac{2}{x-2} + \frac{0x+1}{x^2+1}\right)dx$$

$$= \int_0^1\left(\frac{2}{x-2} + \frac{1}{x^2+1}\right)dx$$

$$= \left(2\ln|x-2| + \arctan(x)\right)\Big|_0^1$$

$$= \left(2\ln|1-2| + \arctan(1)\right) - \left(2\ln|0-2| + \arctan(0)\right)$$

$$= 2\ln|-1| + \frac{\pi}{4} - 2\ln|-2| + 0$$

$$= 2\ln(1) + \frac{\pi}{4} - 2\ln(2) + 0$$

$$= 0 + \frac{\pi}{4} - 2\ln(2) + 0$$

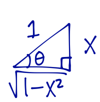
13d. $\int_{e^2}^{e^4} \frac{2}{x\ln(x)}dx = \int_{u=2}^{u=4} \frac{2}{\cancel{x}\cdot u}(\cancel{x}du) = 2\int_{u=2}^{u=4}\frac{1}{u}du = 2\ln|u|\Big|_{u=2}^{u=4} = 2\ln|4| - 2\ln|2| = 2\ln(4) - 2\ln(2)$

$u=\ln(x)$
$\frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

$x=e^4 \Rightarrow u=\ln(e^4)=4$
$x=e^2 \Rightarrow u=\ln(e^2)=2$

13.e $\int\frac{x^3}{\sqrt{1-x^2}}dx = \int\frac{(\sin\theta)^3\cos\theta d\theta}{\sqrt{1-\sin^2\theta}} = \int\frac{\sin^3\theta\cos\theta d\theta}{\sqrt{\cos^2\theta}} = \int\frac{\sin^3\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}d\theta = \int\sin^3\theta d\theta = \int\sin\theta(1-\cos^2\theta)d\theta = -\int(1-u^2)du$

Let $x=\sin\theta$
$dx=\cos\theta d\theta$



$u=\cos\theta$
$\frac{du}{d\theta}=-\sin\theta$

$$= -u + \frac{u^3}{3} + C = -\cos\theta + \frac{\cos^3\theta}{3} + C = -\frac{\sqrt{1-x^2}}{1} + \frac{\left(\frac{\sqrt{1-x^2}}{1}\right)^3}{3} + C.$$

$\leftarrow$ CORRECTION!

14. Summary of information

	$(-\infty,0)$	$(0,1)$	$(1,2)$	$(2,3)$	$(3,4)$	$(4,\infty)$
Sign of f'	+	+	−	−	+	+
behaviour	INC	INC	DEC	DEC	INC.	INC
sign of f''	−	−	−	+	+	−
curvature	CONC. DOWN	CONC. DOWN	CONC. DOWN	CONC. UP	CONC. UP	CONC. DOWN

0   crit# 1   2   crit# 3   VA 4