

- [2] 1. (multiple choice) Which **one** of the following statements is **always** true?
- (A) If a function is continuous at a , then it is differentiable at a .
 - (B) If a function is continuous, then it has an absolute maximum.
 - (C) If f attains its global maximum on a closed interval at point p , then $f'(p) = 0$.
 - (D) If a function is differentiable at a , then it is continuous at a .
 - (E) If $f''(p) = 0$, then f has an inflection point at p .
 - (F) If p is a critical number of a function f , then f has a local maximum or a local minimum at p .
- [2] 2. (multiple choice) Consider the function $f(x) = xe^{3-(x/4)}$. Which one of the following is true?
- (A) $(4, 4e^2)$ is a critical point and a local minimum.
 - (B) $(4, 4e^2)$ is a critical point and a local maximum.
 - (C) $(4, 4e^2)$ is a critical point and an inflection point.
 - (D) $(4, 4e^2)$ is not a critical point but it is an inflection point.
 - (E) $(4, 4e^2)$ is a critical point but not a local extremum and not an inflection point.
 - (F) $(4, 4e^2)$ is not a point on the curve.
- [2] 3. (multiple choice) Suppose $f(x)$ is a continuous function on the interval $[a, b]$. Which **one** of the following statements is **always** true?
- (A) $\int_a^b f(x) dx = \int_b^a f(x) dx$.
 - (B) $\int_a^b f(x) dx = f(b) - f(a)$.
 - (C) $\int_a^b f(x) dx = \sum_{i=1}^n f(x_i)\Delta x$, where n is a positive integer, $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.
 - (D) If $f(x) \leq c$ on $[a, b]$ then $\int_a^b f(x) dx \leq c(b-a)$.

(E) $\int_a^b f(x) dx$ is an antiderivative of f .

(F) $\int_a^b f(x) dx$ is the area of the region bounded by $y = 0$, $y = a$, $y = b$ and $y = f(x)$.

- [2] 4. The following table gives its velocity $v(t)$ of a particle, as measured at regular intervals. Estimate the total displacement of the particle from time $t = 3$ to $t = 6$ s using the trapezoidal rule.

t (s)	3	3.5	4.0	4.5	5.0	5.5	6.0
$v(t)$ (m/s)	7	3	1	-4	-1	5	6

- [2] 5. Find the derivative of $f(x) = \sqrt[3]{\arcsin(e^x)}$.

- [3] 6. Find the equation of the tangent line to the curve defined implicitly by the equation

$$5e^{xy-1} = 6 - xy^2$$

at the point $(1, 1)$.

- [3] 7. If $f(x)$ is a continuous function such that

$$\int_{-2}^5 f(x) dx = 7 \quad \text{and} \quad \int_1^5 f(x) dx = 10,$$

what is $\int_{-2}^1 (2f(x) - 4) dx$?

- [3] 8. Find the linearization $L(x)$ of the function $f(x) = 32x^{-1/2}$ at $x = 4$ and use it to give a linear approximation of $\frac{32}{\sqrt{4.5}}$.

- [4] 9. Find the following limits.

a) $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

b) $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{\sin(x^2)}$

- [2] 10. Find the derivative of

$$A(t) = \int_{\cos(t)}^1 \sqrt{1+x^3} dx.$$

- [4] 11. If 1200 cm² of material is available to make a box with a square base and an open top, what dimensions will give the largest possible volume of the box?
Make a diagram and **label it** carefully.
Justify that your final answer gives a maximum volume.
- [2] 12. We want to use Newton's Method to find the root of the polynomial $p(x) = x^5 - 4x + 2$ that is between 1 and 2, starting with initial approximation $x_1 = 1$. Write down the formula for Newton's Method and find x_2 and x_3 .
- [15] 13. Find the following:
- $\int \tan^5 x \sec x \, dx$
 - $\int t^2 \sin(5t) \, dt$
 - $\int_0^1 \frac{2x^2 + x}{(x-2)(x^2+1)} \, dx$
 - $\int_{e^2}^{e^4} \frac{2}{x \ln(x)} \, dx$
 - $\int \frac{x^3}{\sqrt{1-x^2}} \, dx =$
- [4] 14. Consider a function $y = f(x)$ satisfying the following properties:
- f is a continuous function, except at $x = 4$
 - $f(x)$ is not defined at $x = 4$
 - $f(0) = 1$
 - $\lim_{x \rightarrow \infty} f(x) = 2$
 - $\lim_{x \rightarrow 4^\pm} |f(x)| = \infty$
 - $f'(x) = 0$ at $x = 1$ and $x = 3$
 - $f'(x) > 0$ for $x < 1$, for $3 < x < 4$ and for $x > 4$
 - $f'(x) < 0$ for $1 < x < 3$
 - $f''(x) < 0$ for $x < 2$ and $x > 4$
 - $f''(x) > 0$ for $2 < x < 4$
- State where the function is increasing and decreasing:
 - State where it is concave up and concave down.
 - State the locations of all local minimums or maximums.
 - Sketch the graph of f in the space below. Label the special features on the graph, including: asymptotes, inflection points, critical points and local minimums or maximums.
- [0] 15. Just for fun, what is the definition of a "boring" function?