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Part B: Choose the best answer to each of the following questions.

1. Find $\int e^{2x}\sqrt{1+e^{2x}}\,dx$.

   (a) $\left(1+e^{2x}\right)^{3/2}+C$

   (b) $e^{2x}\left(1+e^{2x}\right)^{3/2}-\left(1+e^{2x}\right)^{5/2}+C$

   (c) $\arctan\left(e^{x}\right)+C$

   (d) *** $\dfrac{\left(1+e^{2x}\right)^{3/2}}{3}+C$

   (e) $\dfrac{e^{2x}}{2}\cdot\dfrac{2\left(1+e^{2x}\right)^{3/2}}{3}+C$

This is a u-sub.

$u = 1+e^{2x}$ $\quad du = e^{2x}\cdot 2\,dx$

$dx = \dfrac{du}{2e^{2x}}$

$$\int e^{2x}\sqrt{1+e^{2x}}\,dx = \int e^{2x}\cdot\sqrt{u}\cdot\frac{du}{2e^{2x}}$$

$$= \int \frac{1}{2}\sqrt{u}\,du = \frac{1}{2}\,\frac{u^{3/2}}{3/2}+C$$

$$= \frac{1}{3}\left(1+e^{2x}\right)^{3/2}+C$$

2. Which of the following is equal to

$$\int \sin^3 x \, dx \ ?$$

(a) $\frac{1}{4}\sin^4 x + C$

(b) *** $\frac{1}{3}\cos^3 x - \cos x + C$

(c) $-\frac{1}{4}\cos^4 x \ + C$

(d) $\sin x - \frac{1}{3}\sin^3 x + C$

(e) $-\sin^2 x \cos x + C$

Note: sin x occurs to an odd power, so our sub is:

$u = \cos x \qquad du = -\sin x \, dx$

$dx = -\frac{du}{\sin x}$

$$\int \sin^3 x \, dx = \int \sin^3 x \cdot \left(-\frac{du}{\sin x}\right)$$

$$= \int -\sin^2 x \cdot du = \int -(1-\cos^2 x) \cdot du$$

$$= \int -(1-u^2)\, du = \int -1 + u^2 \, du$$

$$= -u + \frac{u^3}{3} + C = -\cos x + \frac{1}{3}\cos^3 x + C$$

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3. Using integration by parts transforms the integral

$$\int (x^2 + 2x)\ln(x^3 + 1)\ dx$$

to which of the following integrals?

(a) $\dfrac{x^2 + 2x}{3x^2} - \displaystyle\int (2x + 2)\ln(x^3 + 1)\, dx.$

(b) $(x^2 + 2x)\ln(x^3 + 1) - \displaystyle\int (2x + 2)\ln(x^3 + 1)\, dx.$

(c) $(2x + 2)\ln(x^3 + 1) - \displaystyle\int \frac{(2x + 2)3x^2}{x^3 + 1}\, dx.$

(d) $(x^3 + x^2/2)\ln(x^2 + 1) - \displaystyle\int \frac{(x^3 + x^2/2)2x}{x^2 + 1}\, dx.$

(e) *** $(x^3/3 + x^2)\ln(x^3 + 1) - \displaystyle\int \frac{(x^3/3 + x^2)3x^2}{x^3 + 1}\, dx.$

<u>Note:</u> we can't integrate the log term easily, so this term must be the u.

$u = \ln(x^3+1)$ $\quad dv = x^2+2x$

$du = \dfrac{3x^2}{x^3+1}\,dx$ $\quad v = \dfrac{x^3}{3} + x^2$

$$(x^2+2x)\ln(x^3+1)\,dx = \left(\frac{x^3}{3}+x^2\right)\ln(x^3+1) - \int \frac{\left(\frac{x^3}{3}+x^2\right)\cdot 3x^2}{x^3+1}\,dx$$

4. Which of the following integrals would best be solved by using integration by parts?

(a) $\int \cos^4 x \sin^3 x \, dx.$ → Trig integral = u-sub.

(b) $\int x^3 \sqrt{2x^2+1} \, dx.$ → $\sqrt{\text{square}+1}$ = Trig. sub. (or u-sub. because Outside power is odd)

(c) *** $\int (3x^2 - 2x + 7)e^{2x} \, dx.$

(d) $\int \frac{2x^2+3x-1}{x^3(x^2+1)} \, dx.$

polynomial · non-poly = Integration by Parts

Rational Function = Partial Fraction Decomposition

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5. Which of the following (inverse) trigonometric substitutions would allow you to integrate

$$\int \frac{7x^2}{\sqrt{16-9x^2}}\,dx\ ?$$

identity we want is $1-\sin^2\theta$

(a) ***$x = \frac{4}{3}\sin\theta$.

(b) $x = \frac{3}{4}\sin\theta$.

(c) $x = \frac{3}{4}\tan\theta$.

(d) $x = \frac{7}{3}\sec\theta$.

(e) None of these substitutions will work.

$$16 - 9x^2 = 16 - 16\sin^2\theta$$
$$= 16(1-\sin^2\theta)$$
$$= 16\cos^2\theta$$

compare terms

$$9x^2 = 16\sin^2\theta$$
$$x^2 = \frac{16}{9}\sin^2\theta$$
$$x = \frac{4}{3}\sin\theta$$

6. Which of the following is equal to $$\int \frac{\sqrt{x^2-9}}{x}\,dx\ ?$$

   ← identity we want is $\sec^2\theta - 1$

   (a) $\frac{2}{3}(x^2-9)^{3/2} - \arcsin\left(\frac{x}{3}\right) + C.$

   (b) $-\frac{4\left(x^2-9\right)^{3/2}}{3x^2} + C.$

   (c) $-\frac{2}{3}(x^2-9)^{3/2} + C.$

   (d) $\ln\left|\frac{\sqrt{x^2-9}}{x} - \frac{x}{3}\right| + C.$

   (e) *** $\sqrt{x^2-9} - 3\,\mathrm{arcsec}\left(\frac{x}{3}\right) + C.$

$$x^2 - 9 = 9\sec^2\theta - 9$$
$$= 9(\sec^2\theta - 1)$$
$$= 9\tan^2\theta$$

so $x^2 = 9\sec^2\theta$

$$x = 3\sec\theta\ , \quad dx = 3\sec\theta\tan\theta\,d\theta$$

$$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{\sqrt{9\sec^2\theta - 9}}{\cancel{3\sec\theta}} \cdot \cancel{3\sec\theta}\tan\theta\,d\theta$$

$$= \int \sqrt{9\tan^2\theta}\cdot\tan\theta\,d\theta = \int 3\tan^2\theta\,d\theta$$

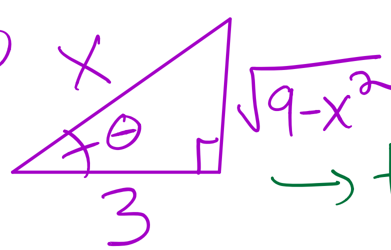
$$= \int 3(\sec^2\theta - 1)\,d\theta = \int 3\sec^2\theta - 3\,d\theta$$

$$= 3\tan\theta - 3\theta + C = 3\cdot\frac{\sqrt{9-x^2}}{3} - 3\,\mathrm{arcsec}\left(\frac{x}{3}\right) + C$$

$\frac{x}{3} = \sec\theta$

$\downarrow$

$\theta = \mathrm{arcsec}\left(\frac{x}{3}\right)$



$\longrightarrow \tan\theta = \frac{opp}{adj} = \frac{\sqrt{9-x^2}}{3}$

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7. What integration technique would allow you to solve the following integral?

$$\int x^2\sqrt{3+7x^2}\;dx\ ?$$

(a) *** Using a Trig substitution.

exactly the thing trig sub is built to do.

(b) Using a u-substitution.

(c) Using integration by parts.

(d) Using a partial fraction decomposition.

(e) Recognizing that the integral is equal to $\cancel{\left(\int x^2\,dx\right)\left(\int\sqrt{3+7x^2}dx\right)}$, where both integrals can be integrated directly

No!!

Not a rational function!

The powers are off, so won't simplify.

The powers don't line up.

$u=3+7x^2,\quad du=14x\,dx$

wrong power of x.

8. Which of the following terms CAN NOT appear as a term in the function that results from computing the following integral?

$$\int \frac{3x^2+2x+1}{x(x+1)^3(x^2+4)^2}\,dx\ ?$$

(a) *** $\frac{A}{x}$.

(b) ~~$\frac{A}{x^2+4}$.~~

(c) ~~$\frac{A}{x+1}$.~~

(d) ~~$A\ln|x|$.~~

(e) ~~$A\ln|x^2+4|$.~~

$$= \int \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} + \frac{Ex+F}{x^2+4} + \frac{Gx+H}{(x^2+4)^2}\,dx$$

$\ln|x|$

$\ln|x+1|$

$(x+1)^{-1}$

$(x+1)^{-2}$

$\ln|x^2+4|$

$\arctan(x)$

$(x^2+4)^{-1}$

arctan, rational function w/ denom $(x^2+4)'$.

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9. After performing a trig substitution, you are left with

$$\int \frac{16}{27} \sin^4 \theta \, d\theta.$$

Which of the following integrals could you have started with?

(a) $\int \frac{9x^4}{\sqrt{4x^2+9}}\,dx$ → tan sub, will look like

$$\frac{\tan^4\theta}{\sec\theta}\cdot\sec^2\theta = \tan^4\theta\cdot\sec\theta$$

(b) *** $\int \frac{9x^4}{\sqrt{4-9x^2}}\,dx$

sin sub, will look like

$$\boxed{\frac{\sin^4\theta}{\cos\theta}\cdot\sin\theta}$$

(c) $\int 9x^4\sqrt{4-9x^2}\,dx$

(d) $\int \frac{9x^4}{\sqrt{4x^2-9}}\,dx$

Also a sin-sub,

$$\sin^4\theta\cdot\cos\theta\cdot\cos\theta$$

sec-sub,

$$\frac{\sec^4\theta}{\tan\theta}\cdot\sec\theta\cdot\tan\theta$$

$$= \sec^5\theta.$$

10. Consider the series:

$$\sum_{n=1}^{\infty} \frac{1}{2n^2 - 4n + 3}$$

Which of the following is a true statement?

(a) The series converges by the Divergence Test. Never true!

(b) The series diverges by the Divergence Test. $\lim_{n\to\infty} \frac{1}{2n^2-4n+3} = 0$, so nope.

(c) The series converges by the Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{2n}$. this series diverges (p-series, p=1).

(d) *** The third term in the sequence of partial sums is $S_3 = \frac{13}{9}$.

(e) The third term in the sequence of partial sums is $S_3 = \frac{1}{9}$. not a partial sum.

$$S_3 = \frac{1}{2-4+3} + \frac{1}{8-8+3} + \frac{1}{18-12+3}$$

$$= 1 + \frac{1}{3} + \frac{1}{9} = \frac{9}{9} + \frac{3}{9} + \frac{1}{9} = \frac{13}{9}$$

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11. The series

$$\sum_{n=1}^{\infty} \frac{3+5^n}{7^n}$$

(a) converges by the Comparison Test with the series $\sum_{i=1}^{\infty} \frac{1}{n^2}$.

(b) converges by the Divergence Test.

(c) diverges by the Divergence Test.

(d) has sum equal to $\frac{13}{4}$.

(e) *** has sum equal to 3.

$$\underbrace{\sum_{n=1}^{\infty} \frac{3+5^n}{7^n}}_{\text{looks like a sum of geometric series.}} = \sum_{n=1}^{\infty} \frac{3}{7^n} + \sum_{n=1}^{\infty} \frac{5^n}{7^n}$$

$a = \frac{3}{7}, \ r = \frac{1}{7}$

$a = \frac{5}{7} \quad r = \frac{5}{7}$

$$\frac{\frac{3}{7}}{1-\frac{1}{7}} = \frac{3/7}{6/7} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{5/7}{1-5/7} = \frac{5/7}{2/7} = \frac{5}{2}$$

$$\frac{1}{2} + \frac{5}{2} = 3.$$

12. Which of the following tests would be the best way to determine the convergence or divergence of the series
$$\sum_{n=1}^{\infty} \frac{2n^2 - n}{3n^4 + 4n}$$
   (a) The Integral test.
   (b) The Divergence Test.
   (c) The Comparison Test with the series $\sum_{n=1}^{\infty} \frac{2}{3n^2}$.
   (d) *** The Limit Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
   (e) It is impossible to tell if the series converges or diverges.

For large n, $\frac{2n^2 \cancel{-n}}{3n^4 \cancel{+4n}} \approx \frac{n^2}{n^4} = \frac{1}{n^2}$

so, should behave like $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which converges (p-series, $p=2>1$)

Note: $\frac{2n^2-n}{3n^4+4n} \leq \frac{2n^2}{3n^4+4n} \leq \frac{2n^2}{3n^4} = \frac{2}{3n^2}$,

so we could also use the comparison test.

Rough Work

Rough Work