Sample Final Exam - ©Jim Stallard

1. Last year, the weekly price of a 4-litre container of milk averaged \$3.89 with a standard deviation of \$0.40. A consumer advocacy group wishes to test if the price of a 4-litre container of milk is higher compared to last year. It is assumed that the standard deviation in the weekly price of milk has stayed the same in 2010.

(a) State the most appropriate null and alternative hypothesis. [1]

(b) To test the null hypothesis stated in (a), the consumer advocacy group randomly selected $n = 35$ weeks out of the year, observing the price of milk on every Monday of each chosen week. The average price of milk was found to be $\overline{X} = \$4.10$. Using this information, find the value of the test statistic produced by the sample. [2]

(c) Testing at $\alpha = 0.05$, does the result in (b) support the null hypothesis in (a)? Ensure your answer is *statistically* justified. [2]

(d) Find the P-value of the result in (b). [1]

(e) In the context of the problem, interpret the meaning of the P-value found in (d). [2]
(Note: An interpretation is NOT a decision rule!)

2. The number of customers who use a drive-thru Automated Teller Machine (or ATM) varies from one hour to the next, with a rate of 7 customers every 15 minutes.

(a) A random variable X is to measure the number of customers who use this drive-thru ATM in a 15-minute interval. What is the probability that between 8 and 10 customers, inclusive, use this ATM in a given 15-minute interval? [2]

(b) What is the probability that the duration of time passing between successive customers using this ATM is between 2 and 5 minutes? [2]

(c) A customer has just driven away from this ATM, and there are no vehicles in the drive-thru ATM line. If at least 2 minutes pass until the next customer enters the drive-thru ATM line, what is the probability that *in total*, no more than four minutes pass until the next customer enters the drive-thru ATM line? [2]

(d) Over the course of a 5-day week, forty-five non-overlapping 15-minute intervals were randomly chosen and the number of customers using the drive-thru ATM is to be observed in each of these 15-minute intervals.

What is the probability that average number of customers using the drive-thru ATM in these forty-five 15-minute intervals will be between 6.5 and 7.75 customers? [2]

3. An commercial real estate economist wishes to investigate the relationship between office rent and commercial vacancies. The economist randomly selected 30 metropolitan areas across Canada and observed office rent in each city (in \$s per square foot of office space) and vacancy rate, which is expressed as the percentage of office space that is vacant. The data is summarized below.

	Vacancy Rate (in %) - X	Office Rent/ft^2 - X
	3	21.70
	11	16.42
	17	14.84
	:	:
	:	:
	2	17.68
	14	19.74
Mean	11.33%	\$17.20
Stand Dev	5.96%	\$3.35

$$\sum_{i=1}^{30} X_i Y_i = 5534.36$$

(a) A scatter plot of the data is provided below. In the space to the right of the plot, comment on the nature of the relationship between a city's commercial vacancy rate and its office rent. [2]

(b) From the data provided, compute the value of the correlation coefficient. [1]

(c) An attempt was made to model the commercial rental rate in a city as a linear function of the city's commercial vacancy rate. A simple linear regression was run producing the following partial MINITAB output:

```
Predictor          Coef
Constant         20.640
Vacancy Rate  -0.30380
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State the least-squares regression which expresses a city's commercial rent expressed as a linear function of its commerical vacancy rate. [2]

(d) Calgary's current commercial vacancy rate is 6.5%. Can you use the model in (b) to predict the commerical rent rates in Calgary? If so, conduct the prediction and interpret the meaning of the value in the context of the data. If not, explain why. [2]

(e) The coefficient of determination was found to be $r^2 = 0.2911$. Interpret the meaning of this statistic in the context of the data. [2]

4. The weight of students who use the main elevators in the Mathematical Sciences Building varies from one student to another in accordance with the normal distribution, the mean weight being 75 kilograms (kgs) and the standard deviation of 10 kilograms.

(a) What is the probability that a randomly chosen student leaving one of the main elevators in the Mathematical Sciences Building weighs at least 68 kilograms to at most 90 kilograms? [2]

(b) Find the 80th percentile, and interpret its meaning in the context of the data. [2]

(c) Twelve students enter one of the main elevators in the Mathematical Sciences Building. Using this information, answer parts (i) and (ii):

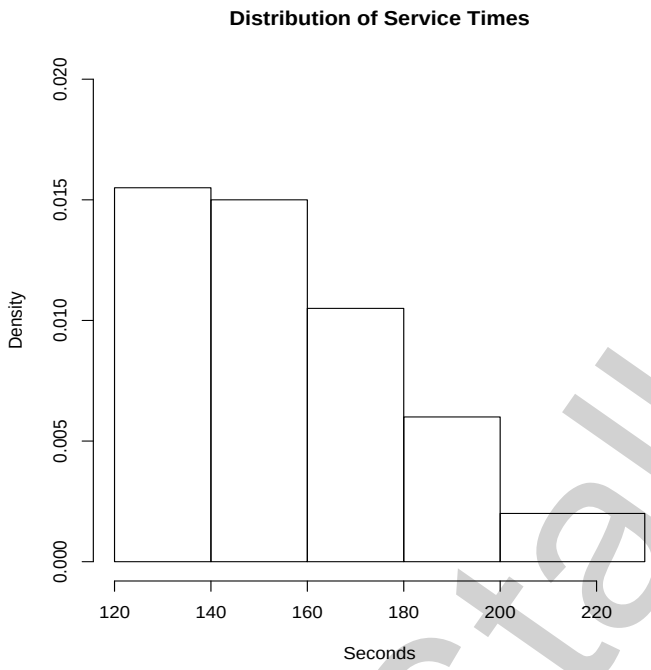
(i) What is the probability that four of these students weigh more than 85 kilograms? [2]

(ii) For safety purposes, the 'maximum capacity' of this elevator is stated as '13 persons, or 1000 kilograms'. What is the probability that this group of twelve students exceed the maximum weight capacity of the elevator? [2]

5. The regional manager of a fast-food restaurant wants to determine how quickly customers are serviced at the fast-food chains drive-thru windows. In the province of Alberta, there are five of these fast-food restaurants. The manager randomly picks 20 customers using the drive-thru window at each of the five restaurants, and has the amount of time passing from the placing of their food order until the receipt of their order, in seconds for $n = 100$.

(a) What type of random sampling was used here? [1]

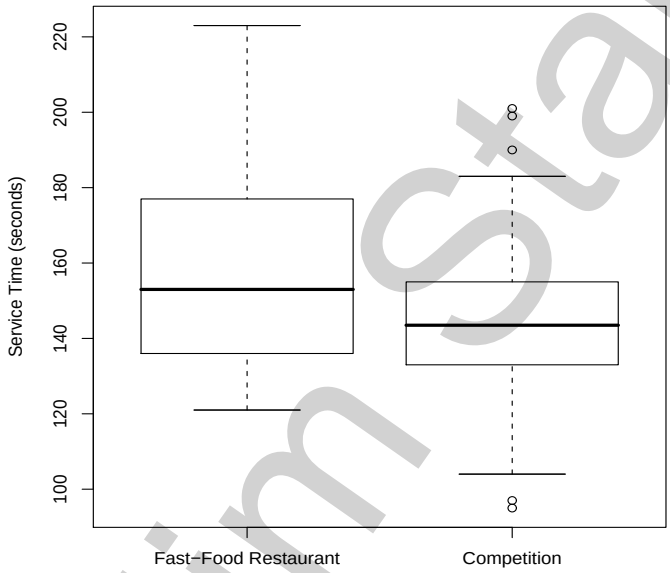
(b) The data is summarized in the histogram below. Comment on the distribution shape in the area to the right of the graph and suggest the most appropriate measure of center. [2]



(c) Some summary statistics were computed: $\overline{X} = 158$ seconds, $\widetilde{X} = 153$ seconds, $S = 25$ seconds. From this sample, what percentage of drive-thru customers will have to wait between 108 seconds and 208 seconds for their order? [2]

(d) Recalling the statistics given in (b): $\overline{X} = 158$, $\widetilde{X} = 153$, $S = 25$, $n = 100$. Find a 95% confidence interval of the mean amount of time a customer is 'in service'. Interpret the meaning of this interval in the context of the data. [3]

(e) Below are two boxplots. The plot on the left-hand side is of the 'drive-thru service times' for the sample taken. The plot on the right-hand side is the 'drive-thru service times' of a sample taken from the fast-food restaurant's main competitor. In the area provided, does there appear to be a difference in the service times between the fast-food restaurant and its competitor? Ensure you justification is statistical. [3]



6. **Part A:** A consumer electronics store wanted to know whether its weekly advertising flyer works. To investigate, the store manager asked people leaving the store two questions: 1. Did you see this week's advertising flyer? 2. Did you make a purchase today? The data was collected and tabulated. The findings were:

- 30% of customers leaving the electronics store left after making a purchase.
- 60% of customers leaving the electronics store had seen the advertising flyer.
- 28% of customers leaving the store neither has seen the advertising flyer nor made a purchase.

(A, i) What is the probability that a randomly chosen customer leaving the electronics store made a purchase or saw the advertising flyer? [2]

(A, ii) If the customer in (A, i) did leave making a purchase, what is the probability he/she did see the advertising flyer? [1]

(A, iii) Are the events 'making a purchase at the store' and 'seeing the store's advertising flyer' independent events? Demonstrate your answer. [1]

Part B: Terry has either a \$1 coin or a \$2 coin in his right-pants-pocket. He is unsure which, but he feels he is three-times more likely to have a \$2 coin in his pocket than a \$1 coin.

He then adds a \$2 coin in his right-pants-pocket, and walks over to the local convenience store (leaving his wallet at home) to buy a newpaper, which costs \$3.50. He fishes one of the two coins out of his right-pants-pocket, and pulls out a \$2 coin. What is the probability that Terry will not have enough money to buy the newspaper? [4]

7. A pollster has been hired by a media outlet to take a random sample of Alberta voters to determine the percentage that would vote for the current premier should a provincial election be held in the near future. A poll conducted a month ago indicated that 20% of Alberta voters would cast a vote for the party that the current premier is the leader of.

(a) If the pollster wished to be 95% confidence that the results of the sample are 'off' by at most four percentage points, how many Alberta voters should the pollster randomly sample? [2]

(b) From the sample size determined in (a), the number of Alberta voters indicating they would vote for the current premier should a provincial election be held in the near future found to be 190. From this result, find a 95% confidence interval for the proportion of Alberta voters that would vote for the current premier. [2]

(c) In the last provincial election, the current premier's party received 42% of the vote. Does the interval in (b) suggest that current premier is *less popular* now compared to the last provincial election? Explain your answer. [2]

End of Final Examination.

JBS/jbs