**CONCORDIA UNIVERSITY**
**Department of Mathematics & Statistics**

| **Course** | **Number** | **Sections** |
|---|---|---|
| Mathematics | 203 | All |
| **Examination** | **Date** | **Duration** |
| Midterm Test | 3 March, 2018 | 1 h 30 min |
| **Special Instructions:** | Only approved calculators are allowed<br>**Show all your work for full marks** | |

**1. (12 marks)**: **(a)** An open rectangular box (no top) has a square base. Given that the height of the box is $x$ and the length $y$ of each side of the base exceeds $x$ by 10 cm, so $y = x + 10$, express the surface area $S$ of the box as a polynomial function of $x$ only (i.e. **not** of $x$ and $y$). in the standard form, i.e. in order of decreasing degree of $x$.

**(b)** Solve for $x$ (find the **exact** values,**do not approximate!**):

$$2\log_4(x) - \log_4(x+3) = 1$$

**(c)** Find the inverse function $f^{-1}(x)$ of $f(x) = \ln(8 - x^3)$ and determine the domain and the range of $f^{-1}(x)$.

**2. (5 marks)** Find **(a)** all horizontal and **(b)** all vertical asymptotes of the graph

$$y = \frac{|4x-1|\,(x^2+2)}{3x^3+12x}$$

**3. (7 marks)** Find the limit or explain why the limit does not exist:

**(a)** $\displaystyle\lim_{x\to -2} \frac{x^2-3x-10}{x^4-16}$

**(b)** $\displaystyle\lim_{x\to 5} \frac{|\,x-5|}{x^2-25}$

**4. (4 marks)** Find the second derivative of the function $f(x) = x^2e^{2x}(x^{-2} + e^{-2x})$ and calculate its **exact** (*not the approximate*!) value $f''(1)$.
(Hint: first simplify the function $f$).

**(continued on the other side)**

**6. (6 marks)** Given the function $f(x) = \sqrt{2x+5}$,

**(a)** Calculate $f'(x)$ using its definition as a limit of difference quotient.

**(b)** Write equation of the tangent line to the curve $y = f(x)$ at the point $(2, 3)$.

**5. (16 marks)** Find the derivatives of the following functions.

(**a**) $f(x) = \dfrac{x + 2\sqrt{x} - 1}{x^{3/2}}$

(**b**) $f(x) = 3^{x^2+1} + (1+2x)^5$

(**c**) $f(x) = \dfrac{\cos^2(x)}{1+\tan x} + [\cos(x)\sec(x)]^2$

(**d**) $f(x) = \sin[\, x\, e^{2x} + \cos(x + e^{2x})]$

**Bonus Question (3 marks)**. Find the $n$-th derivative $f^{(n)}(x)$ if $f(x) = \dfrac{1}{2-x}$ and $n$ is an arbitrary positive integer.