# MATH 1007 B, Tutorial 7

Instructions: Work in teams of 3 or 4. At the end of the tutorial, each team hands in one set of solutions with everybody's name and student number. Do not divide up problems and work on them separately.

**Question 1**. [5 marks] Evaluate the limit of the following sum and show some of your steps.

$$\lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2$$

Hint: First evaluate the sum in terms of $n$ and then take the limit as $n \to \infty$ and also remember that $\sum_{k=1}^{n} k^2 = \frac{n(n-1)(2n+1)}{6}$.

Solution:

$$\lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2 = \lim_{n\to\infty} \frac{n(n-1)(2n+1)}{6n^3} = \frac{2}{6} = \frac{1}{3}$$

**Question 2**. [5 marks] Use the properties of definite integrals and show that the following inequality is true.

$$\int_0^\pi \sqrt{\sin^2 x + 3}\; dx \leq 2\pi$$

Solution: On the interval $[0, \pi]$ the maximum of $\sqrt{\sin^2 x + 3}$ is $\sqrt{\sin^2 \frac{\pi}{2} + 3} = \sqrt{4} = 2$. Hence,

$$\int_0^\pi \sqrt{\sin^2 x + 3} dx \leq 4\pi \leq 2\ (\pi - 0) = 2\pi$$

**Question 3**. Evaluate the following definite integrals. Show some of your steps.

(a) [5] $\int_{-1}^{1} (x^3 - x^2)\; dx$

Solution: $\int_{-1}^{1} (x^3 - x^2) dx = \int_{-1}^{1} x^3 dx - \int_{-1}^{1} x^3\; dx = \frac{x^4}{4}\Big|_{-1}^{1} - \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{4} - \frac{1}{4} - (\frac{1}{3} - \frac{(-1)^3}{3}) = \frac{-2}{3}$.

(b) [5] $\int_0^{\frac{\pi}{2}} \cos x \ dx$

Solution: $\int_0^{\frac{\pi}{2}} \cos x = \sin x\Big|_0^{\pi/2} = \big(\sin\frac{\pi}{2} - \sin 0\big) = 1.$

(c) [5] $\int_1^e \frac{1}{x} dx$

Solution: $\int_1^e \frac{1}{x} dx = \ln|x| \Big|_1^e = \ln e - \ln 1 = 1 - 0 = 1.$