MATH 1007 B

Instructions: Work in teams of 3 or 4. At the end of the tutorial, each team hands in one set of solutions with everybody's name and student number. Do not divide up problems and work on them separately.

Question 1. Find $f'(x)$ for the following functions.

(a) $f(x) = (4x+2)^2(5x^3-2)^3 \to f'(x) = 2(4x+2)(4)(5x^3-2)^3 + 3(5x^3-2)^2(15x^2)(4x+2)^2 = 8(4x+2)(5x^3-2)^3 + 45x^2(5x^3-2)^2(4x+2)^2$

(b) $f(x) = \frac{2x^3-1}{3x^2+2} \to f'(x) = \frac{(6x^2)(3x^2+2)-(6x)(2x^3-1)}{(3x^2+2)^2} = \frac{6x(x^3+2x+1)}{(3x^2+2)^2}$

(c) $f(x) = 3\sin(\sqrt{x}) \to f'(x) = 3\cos(\sqrt{x})(\frac{1}{2\sqrt{x}}) = (\frac{3\cos(\sqrt{x})}{2\sqrt{x}})$

(d) $f(x) = \sqrt[3]{(x^3+2x-1)^2} \to f'(x) = \frac{2}{3}(x^3+2x-1)^{\frac{-1}{3}}(3x^2+2) = \frac{2(3x^2+2)}{3\sqrt[3]{x^3+2x-1}}$

(e) $f(x) = \tan x + \cot(2x) \to f'(x) = \sec^2(x) - 2\csc^2(2x)$

(f) $f(x) = \frac{\sin x}{\tan x} \Rightarrow f(x) = \cos x \to f'(x) = -\sin x$

Question 2. Evaluate $\frac{dy}{dx}|_{x=a}$ if

(a) [3] $y = x^5 - 3x^2 + 3, \quad a = 1 \to \dfrac{dy}{dx} = 5x^4 - 6x \Rightarrow \dfrac{dy}{dx}|_{x=1} = 5 - 6 = -1$

(b) [3] $y = \sin(\sin x), \quad a = \pi \to \dfrac{dy}{dx} = \cos(\sin x)\cos x \Rightarrow \dfrac{dy}{dx}|_{x=\pi} = (1)(-1) = -1$

(c) [3] $y = \tan(x) - x, \quad a = \dfrac{\pi}{4} \to \dfrac{dy}{dx} = 1 + \tan^2(x) - 1 = \tan^2 x \Rightarrow \dfrac{dy}{dx}|_{x=\frac{\pi}{4}} = 1$

Question 3. The distance to the origin of a particle moving on the real line, after t seconds from its departure is given by

$$x(t) = -0.1t^2 + 10t.$$

(a) [2] Find the speed of the particle 5 seconds after its departure from the origin. Determine whether the particle is moving towards or away from the origin at that time.
Solution: $v(t) = x'(t) = -0.2t + 10 \Rightarrow v(5) = 9$. Since $v(5)$ is positive, the particle is moving away from the origin.

(b) [2] Answer the questions in part (a), for 1 minute after the starting time.
Solution: $v(60) = -2$. Since $v(60)$ is negative, the particle is moving towards the origin.