MATH 1007 B, Tutorial 3

Instructions: Work in teams of 3 or 4. At the end of the tutorial, each team hands in one set of solutions with everybody's name and student number. Do not divide up problems and work on them separately.

Question 1. (a) [3] Find the interval on which the function $y = f(x) = \dfrac{1}{x^3 + 125}$ is one-to-one. (Do not forget to obtain the domain of f first)

(b) [4] Find the inverse of f, i.e. find f^{-1}, on the interval you obtained in Part (a).
Advise: Check your answer. In other words after obtaining f^{-1} make sure the following two relations hold true.

$$(fof^{-1})(x) = x \; and \; (f^{-1}of)(x) = x.$$

Solution: Solve this equation to obtain x in terms of y as follows.

$$y = \frac{1}{x^3 + 125} \to \frac{1}{y} - 125 = x^3 \to x = \big(\frac{1}{y} - 125\big)^{1/3}$$

Now exchange x and y to get $y = \big(\frac{1}{x} - 125\big)^{1/3}$. This means that $f^{-1}(x) = \big(\frac{1}{x} - 125\big)^{1/3}$.

Question 2. (a) [3] Find the interval(s) on which the function $y = e^{x^2}$ is invertible (one-to-one).
Solution: $y = e^{x^2}$ is one-to-one (and as a result invertible) either on $[0, \infty)$ or on $(-\infty, 0]$.
(b) [5] Find the inverse of $y = e^{x^2}$ on $[0, \infty)$.
Hint: Recall the relation between $\ln x$ and e^x.

Solution:

$$y = e^{x^2} \to x^2 = \ln y \to x = \sqrt{\ln y}$$

Now Exchange x and y to get the inverse of as $y = \sqrt{\ln x}$.
Question 3. [6] Find the value of x such that

$$\frac{\pi}{3} = \sin^{-1} x^2$$

Hint: Remember the definition of $\sin^{-1}$ and that $\sin \frac{\pi}{3} = \frac{1}{2}$.

Question 4. [4] A car starts moving from a spot. The distance (in meters) of this car from the start point at second t is $y = 4t + 2$. Find the average speed of this car between the 5th and the 8th second.

Solution: $\dfrac{\Delta y}{\Delta t} = \dfrac{4 * 8 + 2 - (4 * 5 + 2)}{8 - 5} = \dfrac{12}{3} = 4$