

LAST NAME: _____

ASSIGNED CLASS NUMBER _____

- 1 Nuclear astrophysics proposes that all the elements heavier than iron are formed in supernova explosions ending the lives of massive stars. Assume that at the time of the explosion the amounts of ^{235}U and ^{238}U were equal. How long ago (to the nearest million years) did the star(s) explode that released the elements that formed our Earth? The present $^{235}\text{U}/^{238}\text{U}$ ratio is 0.007 25. The half-lives of ^{235}U and ^{238}U are 0.704×10^9 yr and 4.47×10^9 yr.

$$\text{We have } N_{235} = N_{0, 235} e^{-\lambda_{235} t} \text{ and } N_{238} = N_{0, 238} e^{-\lambda_{238} t} \quad \frac{N_{235}}{N_{238}} = 0.007 25 = e^{(-\ln 2)t/T_{h, 235} + (\ln 2)t/T_{h, 238}}$$

$$\text{Taking logarithms, } -4.93 = \left(-\frac{\ln 2}{0.704 \times 10^9 \text{ yr}} + \frac{\ln 2}{4.47 \times 10^9 \text{ yr}} \right) t \quad -4.93 = \left(-\frac{1}{0.704 \times 10^9 \text{ yr}} + \frac{1}{4.47 \times 10^9 \text{ yr}} \right) (\ln 2) t$$

$$t = \frac{-4.93}{(-1.20 \times 10^{-9} \text{ yr}^{-1}) \ln 2} = \boxed{5.94 \times 10^9 \text{ yr}}$$

- 2 A star ending its life with a mass of two times the mass of the Sun is expected to collapse, combining its protons and electrons to form a neutron star. Such a star could be thought of as a gigantic atomic nucleus. If a star of mass $2 \times 1.99 \times 10^{30}$ kg collapsed into neutrons ($m_n = 1.67 \times 10^{-27}$ kg), what would its radius be? (Assume that $r = r_0 A^{1/3}$.)

$$\text{The number of nucleons in a star of two solar masses is } A = \frac{2(1.99 \times 10^{30} \text{ kg})}{1.67 \times 10^{-27} \text{ kg/nucleon}} = 2.38 \times 10^{57} \text{ nucleons}$$

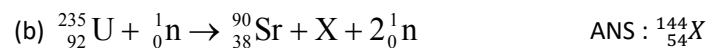
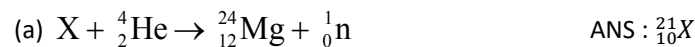
$$\text{Therefore: } r = r_0 A^{1/3} = (1.20 \times 10^{-15} \text{ m}) (2.38 \times 10^{57})^{1/3} = \boxed{16.0 \text{ km}}$$

- 3 The half-life of ^{131}I is 8.04 days. On a certain day, the activity of an iodine-131 sample is 6.40 mCi. What is its activity 40.2 days later?

$$R = R_0 e^{-\lambda t} = (6.40 \text{ mCi}) e^{-(\ln 2/8.04 \text{ d})(40.2 \text{ d})} = (6.40 \text{ mCi}) (e^{-\ln 2})^5 = (6.40 \text{ mCi}) \left(\frac{1}{2^5} \right) = \boxed{0.200 \text{ mCi}}$$

- 4 Determine the activity of 1.00 g of ^{60}Co . The half-life of ^{60}Co is 5.27 yr. Activity $R = \lambda N$ we need λ and N
 $\lambda = \frac{\ln 2}{\tau_{1/2}} = \frac{\ln 2}{1.662 \cdot 10^8 \text{ s}} = 4.17 \cdot 10^{-9} \frac{1}{\text{s}}$ and $N = \frac{1 \text{ g}}{60 \text{ g}} \text{ mole} = 1.0033 \cdot 10^{22}$ so that: $R = \lambda N = 4.1839 \cdot 10^{13} \text{ Bq}$
 $R = 1130.8 \text{ Ci}$

- 5 Identify the unknown nuclei and particles X and X' in the following nuclear reactions:



- 6 The radial part of the Schrödinger equation in spherical coordinates is

$$-\frac{\hbar^2}{2m} \left(\frac{d^2 \psi}{dr^2} + \frac{2}{r} \frac{d\psi}{dr} \right) - \frac{k_e e^2}{r} \psi = E \psi$$

a) Show that the 1s wave function for an electron in $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ hydrogen, satisfies the Schrödinger equation.

b) find the probability that the electron is inside of the nucleus ($R=1.2\text{fm}$)
 (use the opposite side of this page for the calculation)

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7 The radial part of the Schrödinger equation in spherical coordinates is

$$-\frac{\hbar^2}{2m} \left(\frac{d^2\psi}{dr^2} + \frac{2}{r} \frac{d\psi}{dr} \right) - \frac{k_e e^2}{r} \psi = E\psi$$

Show that the 1s wave function for an electron $\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ in hydrogen, satisfies the Schrödinger equation.

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \quad \frac{2}{r} \frac{d\psi}{dr} = \frac{-2}{r\sqrt{\pi a_0^3}} e^{-r/a_0} = -\frac{2}{ra_0} \psi$$

$$\frac{d^2\psi}{dr^2} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} = \frac{1}{a_0^2} \psi \text{ so that: } -\frac{2}{2m_e} \left(\frac{1}{a_0^2} - \frac{2}{ra_0} \right) \psi - \frac{e^2}{4\pi \epsilon_0 r} \psi = E\psi$$

But $a_0 = \frac{2(4\pi \epsilon_0)}{m_e e^2}$ so $-\frac{e^2}{8\pi \epsilon_0 a_0} = E$ or $E = -\frac{k_e e^2}{2a_0}$.

This is true, so the Schrödinger equation is satisfied.

8 FILL THE BLANKS

A) List 4 fundamental forces in the from the strongest to the weakest.

ANS:

1strong nuclear 2electromagnetic 3weak nuclear 4 gravitational

B) In your own words explain the essence of the Noether's Theorem?

Each conservation law corresponds to the invariance under certain symmetry operation

C) Existence of the Gravitational Waves have been just recently confirmed by Laser Interferometer Gravitational Observatories. They are essentially very large: Michelson Interferometers.

D) It is well established fact that stars evolve. Given the mass of our Sun and the current state of knowledge about stellar nucleosynthesis, what is the most probable final stage of the Sun's Evolution?

ANS: White Dwarf (eventually Brown Dwarf)

8 Write down the most fundamental formulas relevant to the material tested in this assignment.