

## MATH 1007 B, Tutorial 2 (Solution)

Instructions: Work in teams of 3 or 4. At the end of the tutorial, each team hands in one set of solutions with everybody's name and student number. Do not divide up problems and work on them separately.

**Question 1.** [6] Let  $f(x) = \frac{1}{x^3}$ .

(a) Find  $(f \circ f)(x)$ .

Solution:  $(f \circ f)(x) = (x^{-3})^{-3} = x^9$

(b) Find  $(f \circ f \circ f)(x)$ .

Solution:  $(f \circ f \circ f)(x) = (x^9)^{-3} = x^{-27}$

**Question 2.** [4] Let  $f(x) = \sqrt{\frac{1}{x} + 10}$  and  $g(x) = \frac{1}{x^2 - 10}$ .

(a) Find  $(f \circ g)(x)$

Solution:  $(f \circ g)(x) = f\left(\frac{1}{x^2 - 10}\right) = x$

(b) Find  $(g \circ f)(x)$

Solution:  $(g \circ f)(x) = g\left(\sqrt{\frac{1}{x} + 10}\right) = x$

**Question 3.** [6] Let  $y = \sqrt{3x - x^2}$ . Write the equation after the following three transformations: First, 4 units to the left, then, reflection about x-axis, then, down 1 units.

Solution: 4 units to the left gives  $y = \sqrt{3(x + 4) - (x + 4)^2}$   
Reflecting about the  $x$ -axis means  $y = -\sqrt{3(x + 4) - (x + 4)^2}$   
Down 1 unit  $y = -\sqrt{3(x + 4) - (x + 4)^2} - 1$

**Question 4.** [3] Let  $f(x) = \sin x$  on  $[0, 2\pi]$ . Also let  $g(x) = x^8$ . Is  $(f \circ g)(x)$  odd or even? Why?

Solution: It is Even. Because  $(f \circ g)(x) = \sin x^8$  and  $\sin(-x)^8 = \sin x^8$ .

Remember that  $\sin x$  is an odd function

**Question 5.** [6] Find all values of  $x$  in the interval  $[0, \pi]$  such that

$$\sqrt{3} \cos x - 2 \sin x \cos x = 0$$

Hint: It will be helpful if you draw the trigonometric circle. Also note that  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ .

] Solution:  $\cos x(\sqrt{3} - 2 \sin x) = 0$  and this means either  $\cos x = 0$  or  $\sin x = \frac{\sqrt{3}}{2}$ .  
But  $\cos x = 0$  gives  $x = \frac{\pi}{2}$  (remember that  $0 \leq x \leq \pi$ )

And  $\sin x = \frac{\sqrt{3}}{2}$  gives  $x = \frac{\pi}{3}, x = \pi - \frac{\pi}{3}$ .

so the values of  $x$  we were looking for are  $x = \frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$ .