MATH 1007 B, TEST 4
Time: 50 Minutes, Total Mark: 25

Name: Student #:

Question 1. Find the derivative of the following functions: (Show some steps of your work)

(a) [5] $\frac{d}{dx}(\sin^{-1} x)^x$,
where $\sin^{-1} x$ is the inverse of $\sin x$.

Solution: $\frac{d}{dx}(\sin^{-1} x)^x = \frac{d}{dx} e^{x \log \sin^{-1} x} = \left(\sin^{-1} x\right)^x \left(\log \sin^{-1} x + \frac{x}{\sin^{-1}(x)\sqrt{1-x^2}}\right)$

(b) [4] $\frac{d}{dx}(x^2 e^x \sin x)$.

Solution:

$$\begin{aligned}\frac{d}{dx}(x^2 e^x \sin x) &= 2x(e^x \sin x) + x^2(e^x \sin x + e^x \cos x) \\ &= 2xe^x \sin x + x^2 \cos x e^x + x^2 e^x \sin x\end{aligned}$$

Question 2. [4] Find the global maximum of the function $f(x) = x^4 - 2x^2 + 2$ on $[-1, 1]$. At what value of x the global maximum is obtained?

Solution: $f'(x) = 4x(x^2 - 1) = 0 \rightarrow x = -1,\ x = 0,\ x = 1$
$f(-1) = 1,\ f(0) = 2,\ f(1) = 1$. Hence, the global maximum on $[-1, 1]$ is $f(0) = 2$ and it is obtained at $x = 0$.

Question 3. [6] Find the critical point(s) of the function $f(x) = x \log x^x - x^2$.

Solution: Observe that

$$x \log x^x - x^2 = x^2 \log x - x^2 = x^2(\log x - 1)$$

$$f'(x) = x(2\log x - 1) = 0 \to x = e^{\frac{1}{2}}$$

Note that $x = 0$ is not in the domain of f. Hence at $x = \sqrt{e}$ the function f has a critical point.

Question 4. [6] Find the linear approximation to $f(x) = (x+1)^{\frac{1}{3}}$ near $x = 0$ to approximate $(1.03)^{\frac{1}{3}}$. (You do not need to simplify your final answer).
Remember that the linear approximation to $f(x)$ near $x = a$ is $L(x) = f(a) + f'(a)(x-a)$.

Solution: $f'(x) = \dfrac{1}{3(x+1)^{\frac{2}{3}}} \to f'(0) = \dfrac{1}{3}$. Also $f(0) = 1$. Now the approximating line is $L(x) = 1 + \frac{x}{3}$. So, $(1.03)^{\frac{1}{3}} \approx 1 + \dfrac{0.03}{3}$ the solution is complete now.
For the that sake of curiosity our approximation is $(1.03)^{\frac{1}{3}} \approx 1.01$.