MATH 1007 B, TEST 3
Time: 50 Minutes, Total Mark: 25

Name: Student #:

Question 1. Evaluate the following limits by showing steps of your answer:

(a) [3] $\lim_{x\to-\infty} \dfrac{18x^4+x^2}{2x^4-5x^3-18x-4}$.

Solution: $\lim_{x\to-\infty} \dfrac{18x^4+x^2}{2x^4-5x^3-18x-4} = \lim_{x\to-\infty} \dfrac{18+\frac{1}{x^2}}{2-\frac{5}{x}+\frac{18}{x^3}-\frac{4}{x^4}} = \dfrac{18}{2} = 9$

(b) [2] $\lim_{x\to\infty} \left(\dfrac{-10x^{10}}{6x^{11}-6}\right)^{\frac{1}{3}}$.

Solution: $\lim_{x\to\infty} \left(\dfrac{-10x^{10}}{6x^{11}-6}\right)^{\frac{1}{3}} = \left(\lim_{x\to\infty} \dfrac{\frac{-10}{x}}{6-\frac{6}{x^{10}}}\right)^{\frac{1}{3}} = 0$

(c) [3] $\lim_{x\to-\infty} \dfrac{e^{-x}}{e^{-2x}}$.

Solution: $\lim_{x\to-\infty} \dfrac{e^{-x}}{e^{-2x}} = \lim_{x\to-\infty} e^{x} = 0$

Question 2. [6] Find the horizontal and vertical asymptotes of the function $f(x) = \dfrac{x^2+4}{x^2-2x+1}$.

Solution: Horizontal asymptote is $y = 1$ since $\lim_{x\to-\infty} \dfrac{x^2+4}{x^2-2x+1} = 1$ and $\lim_{x\to\infty} \dfrac{x^2+4}{x^2-2x+1} = 1$.

The candidate(s) for vertical asymptotes are obtained from setting $x^2-2x+1=0$ this implies that $x = 1$ is the candidate. Now observe that

$$\lim_{x\to1^+} \frac{x^2+4}{x^2-2x+1} = \infty$$

and

$$\lim_{x\to1^-} \frac{x^2+4}{x^2-2x+1} = \infty$$

So $x = 1$ the (only) vertical asymptote of $f(x) = \frac{x^2+4}{x^2-2x+1}$.

Question 3. [5] Find the slop of the tangent line to curve of the function $f(x) = x^2 - 1$ at $x = 3$.

Hint: The slop m of the tangent line at the point x_0 is $m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$.

Solution:

$$m = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{(3+h)^2 - 1 - 3^2 + 1}{h} = \lim_{h \to 0} \frac{h^2 + 6h}{h} = 6$$

Question 4. [6] Is the function $f(x) = \begin{cases} x^2, & x \geq 0; \\ 2 - x^3, & x < 0. \end{cases}$ differentiable at $x = 0$? Explain your answer.

Solution: The answer is NEGATIVE. One way of showing that f is not differentiable at $x = 0$ is to show

$$\lim_{x \to 0^+} \frac{f(x+h) - f(x)}{h} \neq \lim_{x \to 0^-} \frac{f(x+h) - f(x)}{h}.$$

The other way (better way) is to observe that f is not continuous at $x = 0$ since

$$\lim_{x \to 0^+} f(x) = 0 \ \& \ \lim_{x \to 0^-} f(x) = 2.$$

Now since f is not continuous at $x = 0$ so it is NOT differentiable at $x = 0$ either. (See the theorem in Page 125 of your book)