MATH 1007 B, TEST 2
Time: 50 Minutes, Total Mark: 25

Name: Student #:

Question 1. For each one of the following 3 parts of this question Show part of your solution.

(a) [2] Find $\lim_{x\to 0} 2\sec x$.
Hint: Remember that $\cos 0 = 1$.

Solution: $\lim_{x\to 0} 2\sec x = \lim_{x\to 0} \frac{2}{\cos x} = \frac{2}{1} = 2$.

(b) [3] Find $\lim_{x\to\pi} \sqrt{\frac{\sin(x+\pi)}{x^3+1}}$
Hint: $\sin 2\pi = 0$

Solution: $\lim_{x\to\pi} \sqrt{\frac{\sin(x+\pi)}{x^3+1}} = \sqrt{\frac{\sin 2\pi}{\pi^3+1}} = \sqrt{\frac{0}{\pi^3+1}} = 0$.

(c) [4] Find $\lim_{x\to 0} \left(\frac{x^3+x^2+x}{-8x^6-8x}\right)^{\frac{1}{3}}$.

Solution: $\lim_{x\to 0} \left(\frac{x^3+x^2+x}{-8x^6-8x}\right)^{\frac{1}{3}} = \lim_{x\to 0} \left(\frac{x^2+x+1}{-8x^5-8}\right)^{\frac{1}{3}} = \left(\frac{1}{-8}\right)^{\frac{1}{3}} = \frac{-1}{2}$.

Question 2. Suppose that $\lim_{x\to -1} f(x) = 9$ and $\lim_{x\to -1} g(x) = -3$.

(a) [4] Find $\lim_{x\to -1} (\sqrt{f(x)} + \frac{g^2(x)}{f(x)-8})$. Show part of your solution.

Solution: $\lim_{x\to -1} (\sqrt{f(x)} + \frac{g^2(x)}{f(x)-8}) = \lim_{x\to -1} \sqrt{f(x)} + \frac{\lim_{x\to -1} g^2(x)}{\lim_{x\to -1} f(x) - 8} = 3 + \frac{9}{9-8}$.

$(b)[4] Find$ $\lim_{x\to -1} (\frac{f(x)}{f(x)+g(x)})^2$. Show part of your solution.

Solution: $\lim_{x\to -1} (\frac{f(x)}{f(x)+g(x)})^2 = \left(\frac{\lim_{x\to -1} f(x)}{\lim_{x\to -1} f(x) + \lim_{x\to -1} g(x)}\right)^2 = (\frac{9}{6})^2 = \frac{9}{4}$.

Question 3. For the following two parts of this question show part of your solution.

(a) [4] Suppose that for each x we have $\sqrt{x^2 + \frac{x}{2}} \leq f(x) \leq \sqrt{x^3 + \frac{1}{2x}}$.

Find $\lim_{x \to 1} f^2(x)$.

Hint: First use the sandwich theorem for $f(x)$.

Solution: First we find limit of $f(x)$ using the sandwich theorem as follows:
$\lim_{x \to 1} \sqrt{x^2 + \frac{x}{2}} = \sqrt{1^2 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$.
Also $\lim_{x \to 1} \sqrt{x^3 + \frac{1}{2x}} = \sqrt{1^3 + \frac{1}{2.1}} = \sqrt{\frac{3}{2}}$. So, $\lim_{x \to 1} f(x) = \sqrt{\frac{3}{2}}$ and as a result $\lim_{x \to 1} f^2(x) = \frac{3}{2}$.

(b) [4] Suppose that for each x we have $\tan x \leq f(x) \leq \cot(x + \frac{\pi}{2})$. Find $\lim_{x \to 0} f(x)$.

Hint: Remember the dentition tan and cot and use the sandwich theorem. Also, remember that $\sin \frac{\pi}{2} = 1$ and $\cos \frac{\pi}{2} = 0$.

Solution: Observe that $\lim_{x \to 0} \tan x = \frac{0}{1} = 0$
Also $\lim_{x \to 0} \cot(x + \frac{\pi}{2}) = \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} = \frac{0}{1} = 0$ and as a result $\lim_{x \to 0} f(x) = 0$.