SOLUTION

1. For the network shown in Figure 1 ,

(a) write the **Mesh equations** using R_1, R_2, R_3, R_4, ,v_{s1} and v_{s2}.. [Use the mesh currents shown]

(b) For the component values R_1=5 kΩ, R_2=10 kΩ, R_3=15 kΩ, R_4 = 20 kΩ, v_{s1} = 20 volts, v_{s2} = 30 volts, determine the branch currents i_1, i_2, i_3 and i_4 . (8 marks)

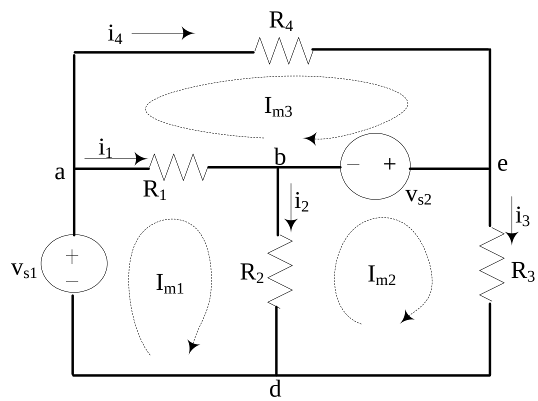

Fig 1.

Solution: (a) $(R_1+R_2)\,I_{m1} - R_2 I_{m2} - R_1 I_{m3} = v_{s1}$

$$-R_1 I_{m1} + (R_2+R_3)\,I_{m2} = v_{s2}$$

$$-R_1 I_{m1} + (R_1+R_4)\,I_{m3} = -\,v_{s2}$$

(b) The equations , with the values given are (currents in mA)

$$15\,I_{m1} - 10\,I_{m2} - 5\,I_{m3} = 20$$

$$-10\,I_{m1} + 25\,I_{m2} = 30$$

$$-5\,I_{m1} + 25\,I_{m3} = -\,30$$

Solving, I_{m1} =13/5 = **2.6 mA** , I_{m2} = 56/25= **2.24 mA**

I_{m3} = – 17/25 = **0.68 mA**

i_1= $I_{m1} - I_{m3}$ = 82/25 = **3.28 mA** , i_2= $I_{m1} - I_{m2}$ = 9/25= **0.36 mA ,**

i_3 = I_{m2} = 56/25 = **2.24 mA** , and i_4= I_{m3} = – 17/25 = **0.68 mA**

2. For the circuit shown in Fig.2,
   (a) obtain the Thevenin Equivalent Circuit across a-b .
   (b) determine the maximum power P_{max} .which can be dissipated in R_L.
   [Hint: For maximum power to be dissipated in R_L, $R_L = R_{Th}$. (8 marks)

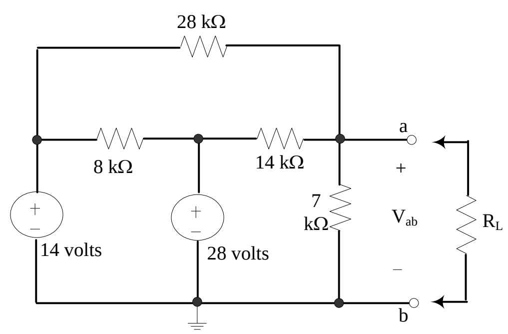

Fig 2.

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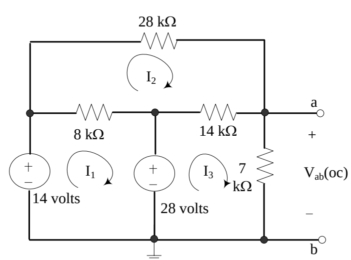
<u>Solution:</u> (a) Using mesh analysis



$8 I_1 - 8I_2 = 14\text{-}28 = -14$, $50 I_2 - 8I_1 - 14I_3 = 0$

and $21 I_3 - 14I_2 = 28$

That is

8	-8	0	:	-14
-8	50	-14	:	0
0	-14	21	:	28

Solving for I_3, $I_3 = \{[8 (1400) – 0)] - (-8) [(-8)(28)-196]\}$

$/ \{8 [1050 – 196)] - (-8) (-8)(21)\} = 7840/5488$ mA

$V_{ab}(oc) = 7 I_3 = 54880/5488 =$ <u>**10 volts.**</u> **= V_{Th}**

$R_{Th} = R_{ab}$ (dead) = (7k) P (14k) P (28k) = $(1/7 +1/14 + 1/28)^{-1}$

= 28/ (4 + 2 +1)= 28/7 = <u>**4** kΩ</u>

[The Thevenin Equivalent Circuit is not shown, but can be drawn with the values of V_{Th} & R_{Th} determined above]

(b) For MPT , $R_L = R_{Th} =$ **4 kΩ** ,

and $P_{max} = (V_T)^2/4R_L = (100)/16000$Watts = **6.25** mW

3. An AC source $V_s = 120\angle 0^o$ volts RMS supplies power to the network shown in Fig.3. Determine the current through the capacitor in the form $I\angle\theta^o$ using any suitable method.

(8 marks)

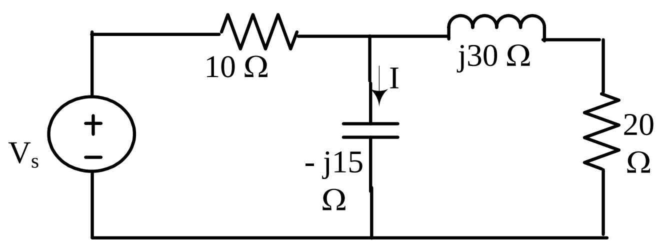


Fig.3

Solution: Two methods are shown below

(1)Using Mesh analysis : the mesh equations (with CW mesh currents I_1 & I_2)are

$((10\text{-}j15)I_1 + j15I_2 = 120$…….(1)

and $j15I_1 + (20+j15)I_2 = 0$ ….(2)

Solving $I_1 = 4.497\angle 49.86^o = 2.899+j3.438$ and

$I_2 = 2.698\angle -77.01^o$ A $= 0.6065 - j\,2.629$, A

Current through C is $I = (I_1 - I_2) = 2.293 + j6.067 =$ **6.486** $\angle$ **69.3**^{o} A

(2) Using Ohm's law & current division:

Total impedance seen by source is $Z = 10 + (20+j\,30)(-j15)/\,(20+j15)$

ie $Z = 10 + 7.2 - j\,20.4 = 17.2 - j20.4 = 26.68\angle -49.87^o$

Total current from source = $I_{tot} = 120/26.68\angle -49.87^o = 4.498\angle 49.87$

By current division, $I = I_{tot}[\,(20+j30)\,/(20+j15)]$

$= 4.498\angle 49.87\;[36.06\angle 56.3^o]\,/25\angle 36.9^o$

$= \mathbf{\underline{6.487} \angle 69.3^o}$ A , as obtained in Method (1)

[The problem can also be solved using other methods available in the current syllabus, such as "voltage division", or using a Thevenin equivalent circuit with respect to Z_c.]

4. The source voltage in the circuit shown in Fig.4. is $v_s(t)= 120 \cos 377t$, volts RMS .The network on the primary side of the ideal transformer is required to supply maximum power P_m to the capacitive load connected to the secondary. Determine
   (a)the value of N required to achieve maximum power transfer
   (b)the value of capacitance C also needed for maximum power transfer
   (8 marks)

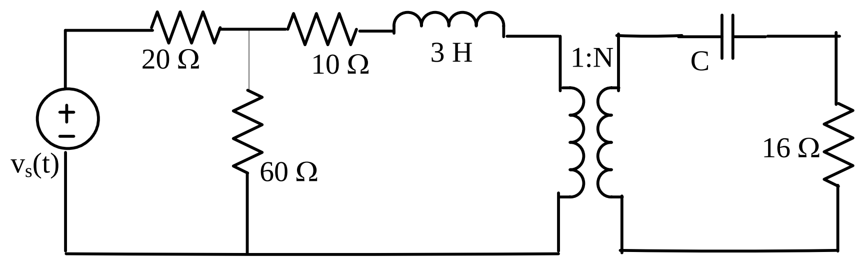


Fig 4.

Solution: (a) The Thevenin Equivalent voltage (across the primary

$V_T = 120\ (60/80) = 90$ vrms

The Thevenin Impedance $Z_T = (20 \text{ P } 60) + 10 + j(377)(3) = 25 + j\ 1131$

The 'primary referred' values of the load elements are $(1/N)^2\ (16)$ and $(1/N)^2\ (1/j377C)$

in series , ie $Z_s' = 16/N^2 - j\ 1/377\ N^2C$

For MPT , $16/N^2 = 25$ or $N = (16/25)^{0.5} =$ **<u>4/5</u>** or N = **<u>0.8</u>**

Also, $1/377\ N^2C = 1131$ or $C = 1/1131(241.28) \approx$ **<u>3.66 μF</u>**

5. For the network shown in Fig.5,

(a) write the basic expression for the input impedance Z_{ab} in terms of R_1, R_2, L ,C and the radian frequency ω, (b) show that $Z_{ab} \approx R_{ab} + j0$, if $R_1 = 1.47$ kΩ , $R_2 = 967\ \Omega$, L = 25H ,C= 25μF and ω = 157 radians/s (6 Marks)

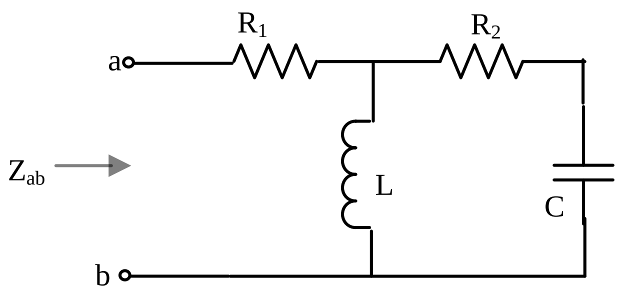


Fig 5.

<u>Solution:</u> (a) $Z_{ab} = R_1 + (j\omega L)(R - j/\omega C) / [R + j(\omega L - 1/\omega C)]$

(b)With the values given, the basic expression in (a) is

$$Z_{ab} = 1470 + (j3925)(967 - j254.78) / [967 + j(3925 - 254.78)]$$

$$= 1470 + (j3925)(967 - j254.78) / [967 + j\,3670.22]$$

$$= 1470 + (3925\angle 90^o)\ 1000\angle -14.76^o) / 3795.47\angle 75.24^o)$$

$$= 1470 + 1034.13\angle 0^o) = 2504.13 + j0 \approx \underline{\mathbf{2.5\ k\Omega}}$$

6. A three-phase, Y-connected balanced load is shown in Fig.6. The reference line-to-line voltage $V_{AB} = 240\angle 0^o$ volts RMS Determine

(a) the power factor (PF) of the load

(b) the <u>magnitude</u> of the line currents (I_L)

(c) the total average power P and the total reactive power Q.

(8 marks)

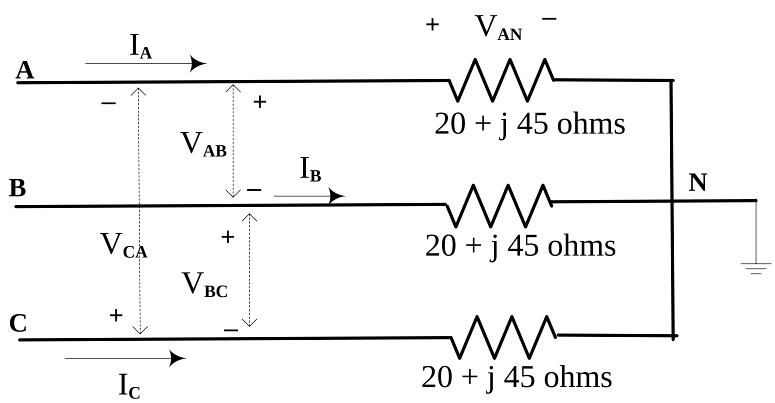


Fig.6.

<u>Solution:</u> (a) The phase impedance $Z = 20+j45 = 49.244\angle 66^o$

ie $\theta = 66^o$ and $PF = \cos\theta = \cos 66^o = 0.407 \approx$ **<u>0.41 lagging</u>**

(b) $I_L = V_p/Z = (V_L/\sqrt{3})/Z = (240/\sqrt{3})/49.244 = 138.564/49.244 = 2.81A$

(c) $P = \sqrt{3}\,V_L I_L \cos\theta = \sqrt{3}\,(240)(2.81)(0.41) =$ **<u>478.92 Watts</u>**

$Q = \sqrt{3}\,V_L I_L \sin\theta = \sqrt{3}\,(240)(2.81)\sin 66^o =$ **<u>1067.11 VAR</u>**

7. Consider the magnetic structure shown in Fig.7.
   (a) Draw the analogous magnetic circuit , clearly showing the MMF, the flux ϕ and the various reluctances
   (b) Determine the current i required to establish a flux $\phi = 2.4(10)^{-4}$Wb in the magnetic circuit. Neglect 'fringing' effects at the air-gap.
   (c) Would a 100% increase in permeability μ, result in a corresponding decrease in the current required to establish the same flux as before ? Justify your answer. (7 marks)

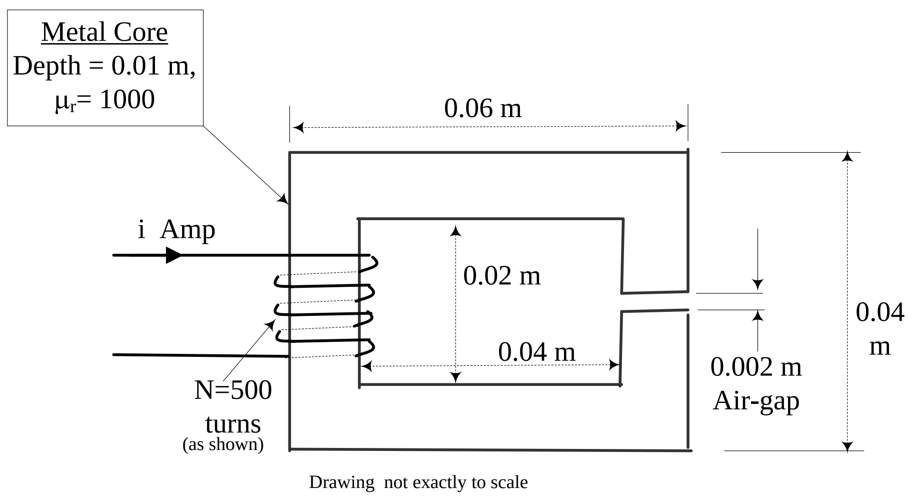
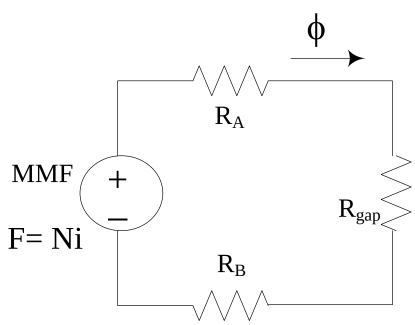


Fig.7.

Soln : (a)The analogous magnetic circuit is shown below



where the MMF is $F = N\,i$ and flux ϕ = MMF/R_m and R_A, R_B and R_{gap} are the reluctances of the 'upper leg' , the 'lower leg' and of the air-gap respectively and $R_A = R_B$.

[Using the Righthand Rule, the flux direction is clockwise in the view of Fig 7.]

(b) Flux ϕ = MMF/$R_{m(total)}$ = Ni / $R_{m(total)}$ where $R_{m(total)} = R_m(\text{metal}) + R_m(\text{air-gap})$

Flux ϕ = Ni / $R_{m(total)}$, hence $i = \phi R_{m(total)} / N$

The reluctances have the form given by

R_m = (mean f lux path length)/ μ (cross-sectional area of path) , (Amp-turn/Wb).

where $\mu = \mu_r \ \mu_o$ and μ_r = relative permeability of the material , $\mu_o = 4\pi(10)^{-7}$ H/m

& $\mu_r = 1$ for air.

R_m (metal) = $R_A + R_B = \ell_{core} / \mu_r A_{core}$ where

ℓ_{core} = 2 (0.05) + 2(0.03) = 0.17 m and $A_{core} = (0.01)(0.01) = 1(10)^{-4}\ m^2$

and $\mu_r = 1000$. Hence R_m (metal)= $\ell_{core} / \mu_r A_{core} = 0.16 / 4\pi(10)^{-7}(1000)(10)^{-4}$

$$R_m \text{ (metal)} = 0.16/1.2566(10)^{-7} = 1.273\ (10)^{6} \text{ A-t/Wb}$$

R_m (air-gap)= $\ell_{gap} / \mu_o A_{gap} = 0.002 / 4\pi(10)^{-7}(10)^{-4} = 0.002/1.2566(10)^{-10}$

$$R_m \text{ (air-gap)} = 1.5913\ (10)^{7} \text{ A-t/Wb}$$

Hence $R_{m(total)} = R_m(\text{metal}) + R_m(\text{air-gap}) = [1.273 + 15.913](10)^6 = 17.186\ (10)^6$ A-t/Wb

$$\text{Flux } \phi = Ni / R_{m(total)}$$

or $i = \phi R_{m(total)} / N = (2.4(10)^{-4})\ [17.186\ (10)^6]\ /500$

$\approx$ **<u>8.25 A</u>**

(c) No. Because the air-gap reluctance is dominant in $R_{m(total)}$. Thus any reduction in R_m(metal) will have small effect on the total reluctance and hence will not result in a greatly decreased value of i.

8 The steadystate equivalent circuit of a DC <u>shunt motor</u> is shown in Fig.8. The motor nameplate gives the following rated values : 440 volts, 1200 RPM, 100 hp [1 hp = 746 Watts] and a full-load efficiency η= 90%.

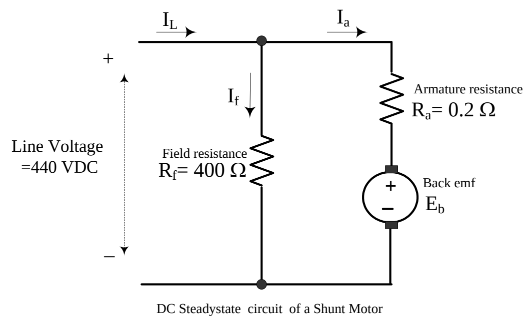


Fig.8

Determine the following :

(a) the total current I_L supplied to the motor

(b)the 'back emf' E_b , at the rated speed ω_1(radians/sec)

(c) the shaft speed ω_2((radians/sec) at which the motor will run when the mechanical load decreases to the level where the armature current is reduced to 100A .Assume the flux per pole to be constant. (7 marks)

<u>Soln</u>: (a) The DC steadystate cicuit of the Shunt motor is shown below

The total line current I_L = P_L /V_L where P_L= Effective power input

P_L= Effective power input = Rated Power /Efficiency = 100 (746)/ 0.9 Watts

ie P_L = 74600/0.9 = 82.9 kW and I_L = P_L /V_L= 82.9/440= **<u>188.4 A</u>**

(b) Using KCL, $I_L = I_f + I_a$ and $I_f = V_L /R_f$ = 440/400 = 1.1 A

ie $I_a = I_L - I_f$ = 188.4 – 1.1= 187.3 A

Using KVL, $440 = I_aR_a + E_b$ or E_b = 440 -(187.3) (0.2) = **<u>402.54 volts</u>**

(c) $E_b= k_a\phi\omega_m$ At the <u>rated values</u> , E_{b1}= 402.54 volts and

ω_{m1} =(2π)/60] (1200) = 125.7 radians/sec, $k_a\phi$ = 402.54/125.7= <u>3.2 volt</u>-sec/radian.

At the reduced load, E_{b2} = 440 - I_aR_a = 440 – (100)(0.2) = 420 volts

and ω_{m2} = E_{b2} / $k_a\phi$ = 420/3.2 = **<u>131.25 r</u>**adians/sec