Elec-275: Winter 2013 Final Exam Solution

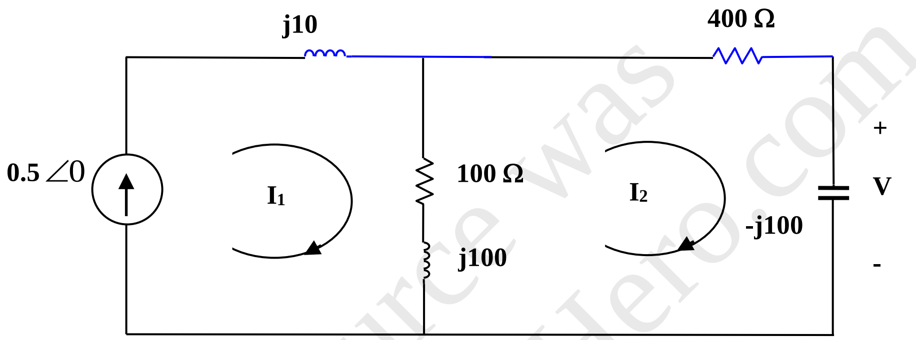
1. (a) For the time-domain circuit of Fig. 1, draw its phasor domain circuit. [Designate I_1, I_2, and V as the phasors of $i_1(t)$, $i_2(t)$, and $v(t)$ respectively]. Draw this phasor circuit.

(b) Using **mesh analysis** on this phasor circuit, determine I_2 and V. Use the meshes shown.

(c) Then write the time domain expressions of $i_2(t)$ and $v(t)$.

Solution:

(a) Phasor circuit:



(b) $I_1 = 0.5\angle 0$

KVL: $500\, I_2 - (100+j100)\, I_1 = 0$; or $500\, I_2 = (100+j100)\times 0.5\angle 0 = 50+j50$.

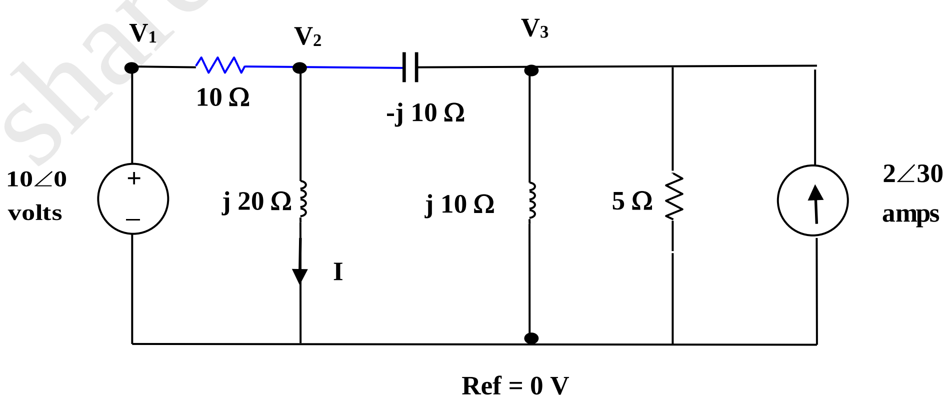
or $\mathbf{I_2} = 0{,}1+j0.1 = 0.1414\angle 45$; $\mathbf{V_2} = -j100\, I_2 = 14.14\angle -45$.

(c) $\boldsymbol{i_2(t)} = 0.1414\cos(1000t+45)$ amps; $\boldsymbol{v_2(t)} = 14.14\cos(10000t-45)$ volts.

2. Using **nodal analysis** in the phasor circuit of Fig.2,

(a) determine the voltages V_2, V_3, and the current I ;

(b) draw the phasor diagrams (plot of phasors in the complex plane) of V_1, V_2, V_3, and I.



Solution:

(a) KCL @ V_2: $\frac{V_2 - 10\angle 0}{10} + \frac{V_2}{j20} + \frac{V_2 - V_3}{-j10} = 0$; or (-1+j2)$V_2$ + 2 V_3 = j 20 ... (1)

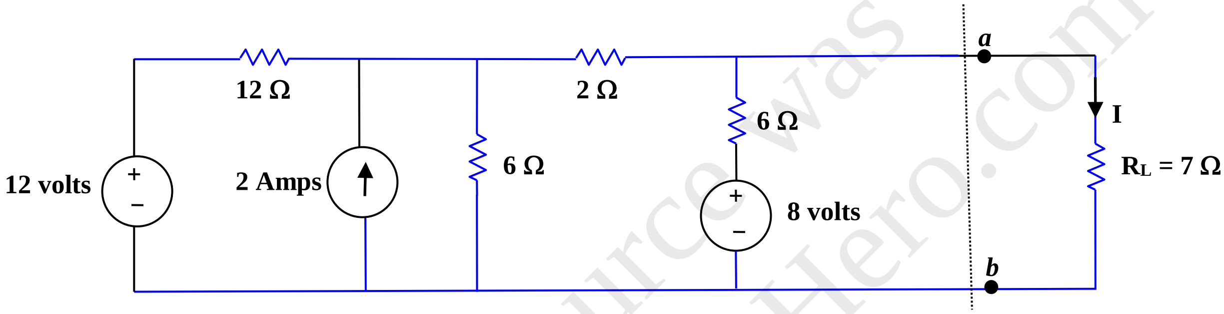
KCL @ V_3 : $\frac{V_3}{j10} + \frac{V_3}{5} + \frac{V_3 - V_2}{-j10} - 2\angle 30 = 0$; or V_2 + j2V_3 = -10+j17.32 ...(2)

From (1) and (2): $\mathbf{V_2}$ = 6.33∠41.6 ; $\mathbf{V_3}$ = 5.3∠24.2; **I** = V_2/(j20) = 0.3165∠-48.4

(b) Phasor diagrams may now be drawn.

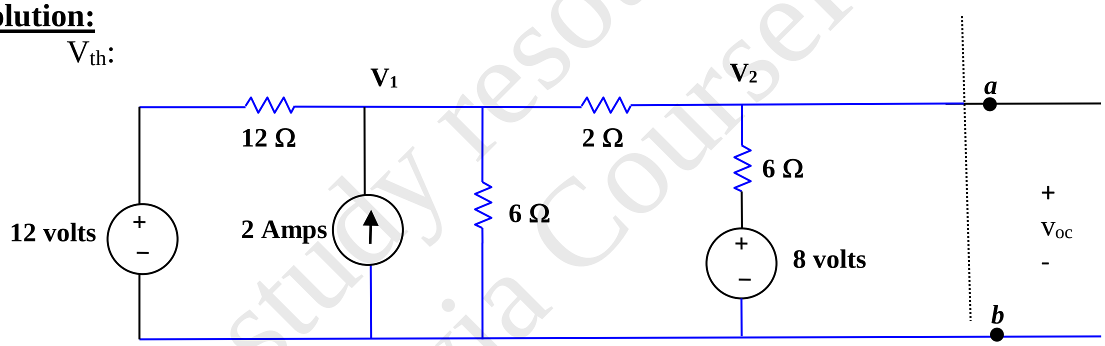
3. (a) Replace the circuit to the left of ***a - b*** of Fig. 3 by its **Thevenin** equivalent. Draw this equivalent circuit.

(b) Using this equivalent circuits, determine the current **I** through the load resistor R_L.



Solution:

V_{th}:

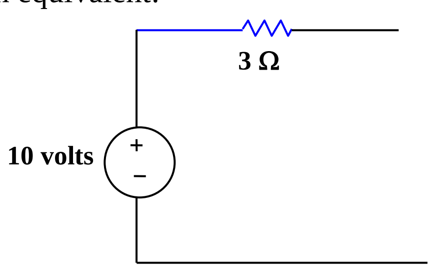


KCL @ V_1: $\frac{V_1 - 12}{12} - 2 + \frac{V_1}{6} + \frac{V_1 - V_2}{2} = 0$; or 9$V_1$ - 6 V_2 = 36 ... (1)

KCL @ V_2 : $\frac{V_2 - V_1}{2} + \frac{V_2 - 8}{6} = 0$; or -3$V_1$ + 4 V_2 = 8 ... (2).

From (1) and (2), V_2 = $\mathbf{V_{th}}$ = 10 volts.

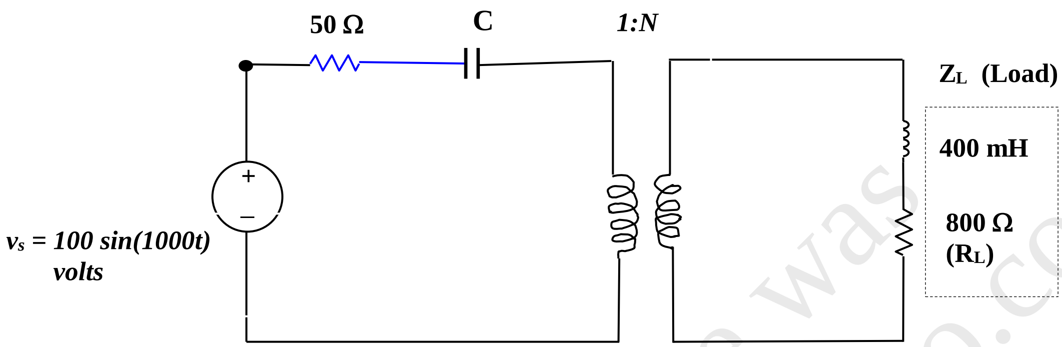
$\mathbf{R_{th}}$ = 3 Ω. Thevenin equivalent:



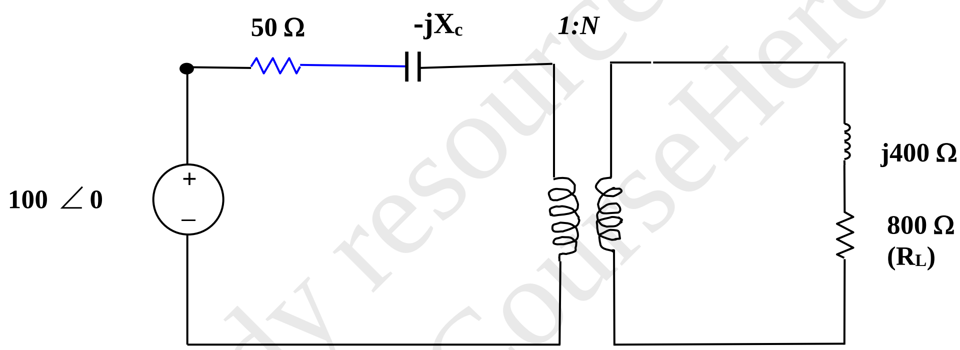
(b) **I** = 1 A.

4. An ideal transformer with a turns ratio of **N** in Fig. 4 is used to match the load $\mathbf{Z_L}$ for maximum power transfer. For that purpose, determine:

   (a) the transformer turns ratio;
   (b) the value of the capacitor **C**;
   (c) the power absorbed by the load.



Solution:



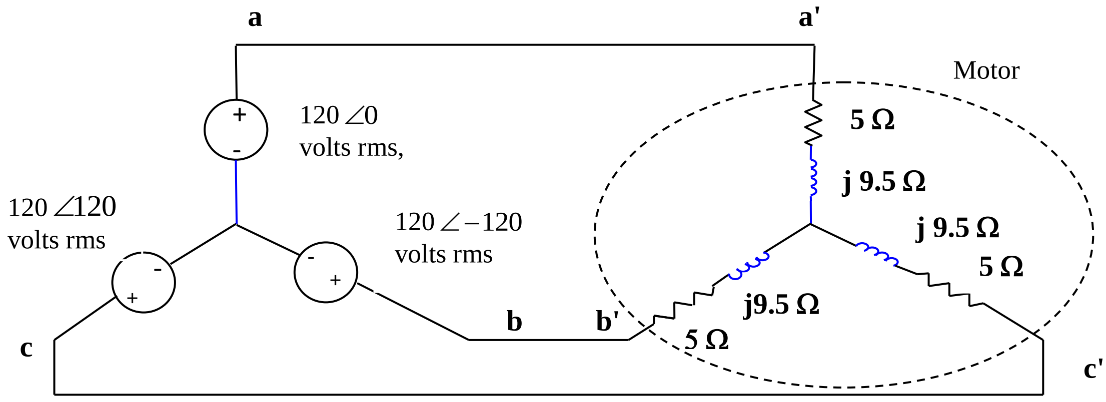
(a): $N^2 = \frac{800}{50} = 16$; $\mathbf{N} = 4$.

(b) $X_c = \frac{400}{16} = 25 = \frac{1}{1000 \times C}$; **C** = 40 μF.

(c) $\mathbf{P} = \frac{(\frac{50}{\sqrt{2}})^2}{50} = 25$ watts.

5. A three-phase 60 Hz power supply is connected to a three-phase motor as shown in Fig. 5. Find:

   (a) the power factor
   (b) the apparent power of the motor
   (c) the real power of the motor
   (d) the reactive power of the motor.



Solution:

Using single phase circuit: $Z_L = 5+j9.5 = 10.735\angle 62.24$

(a) **P.F.** = cos 62.24 = 0.4657.

(b) $S = \frac{V_{rms}^2}{Z^*} = \frac{120^2}{10.735\angle -62.24} = 1341.4\angle 62.24 = 624.7+j\ 1187$

Total power = 3 S = 4024.2 $\angle$ 62.24

Apparent power = 4024.2 VA

(c) **Real power**= 1874.34 W

(d) **Reactive power** = 3561 VAR.

6. For the magnetic circuit of Fig.6:

- Air gap cross sectional area = 2 cm $\times$ 2 cm (for both gaps)
- Air gap lengths:

  l_{g1} = 2 mm
  l_{g2} = 4 mm

- Neglect the reluctance of the magnetic metallic structure (compared to those of the air gaps), as well as the fringing effect.
- The magnetizing coil has 100 turns and carries a current of 0.5 amps.

(a) Determine, for each air gap:

(i) the reluctance **R**;
(ii) the flux **ϕ.**

(b) Find
(i) the flux density **B** for air gap-1 only;
(ii) the field intensity **H** for air gap-1 only.

(c) Find the equivalent reluctance seen by the magnetomotive force **NI**.

Solution:

$A = 4 \times 10^{-4}\ m^2$.

(a) Air gap 1: (i) $\mathbf{R_1} = \dfrac{2\times10^{-3}}{4\pi\times10^{-7}\times4\times10^{-4}} = 3.98\times10^{6}$ A-turns/Wb;

(ii) $\boldsymbol{\phi_1} = \dfrac{NI}{R_1} = \dfrac{50}{3.98\times10^{6}} = 12.566\times10^{-6}$ Wb.

Air gap 2: (i) $\mathbf{R_2} = 7.98\times10^{6}$ A-turns/Wb;

(ii) $\boldsymbol{\phi_2} = 6.283\times10^{-6}$ Wb.

(b) (i) $\mathbf{B} = \dfrac{\phi_1}{A} = \dfrac{12.566\times10^{-6}}{4\times10^{-4}} = 0.0314$ Wb/m^2

(ii) $\mathbf{H} = \dfrac{B}{\mu_0} = \dfrac{0.0314}{4\pi\times10^{-7}} = 25\times10^{3}$ A-turn/m.

(c) $R_{eq} = R_1 \parallel R_2 = 2.65$ A-turn/Wb.

7. (a) $I_f = 240/120 = 2$ A.

At no-load: $I_a = 6 - 2 = 4$A. $E_b = 240 - 0.4\times4 = 238.4 = (K_a\phi)\omega_m$

$= K_a\phi \times \dfrac{2000}{60}\times 2\pi$; or $\mathbf{K_a\phi} = \dfrac{238.4}{2000\times2\pi}\times 60 = 1.138.$

(b) At full-load: $I_a = 50 - 2 = 48$ amps; $\mathbf{E_b} = 240 - 48\times0.4 = 220.8$ volts.

(c) Speed = $\omega_m = \dfrac{220.8}{1.138}\times\dfrac{60}{2\pi} = 1852.35$ rpm.

(d) Torque = $(K_a\phi)\, I_a = 1.138\times48 = 54.637$ N-m

(e) Power= Torque $\times$ speed = $54.637\times\dfrac{1852.35}{60}\times2\pi$ = 10,598.4 W = 14.2 HP.

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