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ECON 3050 Answers to Assignment 1 (Part 2)

Carleton University, Fall 2019

Answers to Assignment 1

Part 2 (Chapters 3 and 4)

(10pts) 3. Consider the timeline below:

Date	**2017**	**2018**	**2019**	**2020**
Cash Flow	–\$235	\$50	\$90	\$150

If the current market rate of interest is 9%, what is the total value of all the cash flows in 2016, 2017, 2018 and 2019 as of today (2018)? (Calculate each one as well as the total value. Show your work.

Answer:

$$FV_{2019}(2017\ cashflow) = -235\,(1.09)^2 = -279.2035 \tag{1}$$

$$FV_{2019}(2018\ cashflow) = 50(1.09) = 54.5 \tag{2}$$

$$PV_{2019}\,(2020\ cashflow) = \frac{150}{(1.09)} = 137.61468 \tag{3}$$

Hence, we can now take the sum of these cashflows since they are now valued at the same period:

$$-279.2035 + 54.5 + 137.61468\ = -87.08882 \tag{4}$$

The total value of this cashflow stream **as of 2019** (including the cash flow received in 2019 of \$90) is then:

$$V_{2019} = 90 - 87.088\,82 = \$2.91118 \approx \$2.9 \tag{5}$$

There are other ways to solve this problem: For example, take the PV of all cash flows as of 2017, and then calculate its future value as of 2019. Alternatively, you could take the future value of all cash flows as of 2020 and then take its present value as of 2019. They all give the same result.

(10pts) 4. You have decided to open a restaurant and need to borrow from your sister. In exchange for a lump sum today, you offer to pay her \$2500 in one year's time, \$3500 in two years' time, \$4500 in three years' time and another \$4500 in 4 years' time. If the interest rate is 6%, what is the lump sum

you can expect to receive from your sister if the net present value (NPV) is to be equal to zero? Show your work. Draw a timeline.

Answer:

Timeline: (From your point of view)

0	1	2	3	4
X	-2500	-3500	-4500	-4500

If the timeline were drawn from the investor's point of view, then the X would have a negative sign and the payments per period would have a positive sign. Both are correct, since the NPV for both cases must be zero.

Solution:

$$
\begin{aligned}
NPV &= X - \frac{2500}{(1+r)} - \frac{3500}{(1+r)^2} - \frac{4500}{(1+r)^3} - \frac{4500}{(1+r)^4} && (6)\\
NPV &= 0 && (7)\\
&\Rightarrow \quad X = \frac{2500}{(1+r)} + \frac{3500}{(1+r)^2} + \frac{4500}{(1+r)^3} + \frac{4500}{(1+r)^4} && (8)\\
\text{Since } r &= 0.06 && (9)\\
X &= \frac{2500}{(1+0.06)} + \frac{3500}{(1+0.06)^2} + \frac{4500}{(1+0.06)^3} + \frac{4500}{(1+0.06)^4} && (10)\\
&= 12816.186 \approx \$12,816.19 && (11)
\end{aligned}
$$

The amount you can receive as a loan today is $\$12,816.19$

Notice that this is not like an annuity because the payments per period are not identical.

(10pts) 5. One ounce of gold is currently worth \$ 1050. A jeweler estimates that the price of gold will rise to \$1075 per ounce in the next year. If she owns 375 ounces of gold, and if the interest rate is 8%, should she sell the gold today and deposit the amount in the bank, or wait until next year to sell the gold? Show your work.

Answer:

First calculate the value of the gold today.

$$CurrentValue = 375 \times 1050 = \$\ 393,750 \qquad (12)$$

Next calculate the value of the gold one year from now when its price will be \$1075 per ounce:

$$FutureValue = 375 \times 1075 = \$\ 403,125 \tag{13}$$

In order to compare the two values, you need to either compute the Future Value (FV) of gold at today's price or calculate the present value (PV) of gold at the price it will have in a year from now, using 8% as the interest rate. Both will lead to the same decision.

You need to provide only one of the two methods, but here are both possibilities:

Now,

$$FV = \ 393750 \times 1.08 = \$\ 425,250 \tag{14}$$

Since $\$\ 425,250 > \$\ 403,125$ she should sell the gold today.

Alternatively,

$$PV = \frac{403125}{1.08} = \$373,263.89 \tag{15}$$

Since the PV of the gold valued at next years price ($\$373,263.89$) is *less* than its current value ($\$\ 393,750$) she should sell the gold today.

(10 pts) 6. You would like to purchase a house worth \$400,000. You can only afford to make a down payment of \$50,000 and need to take out a mortgage of \$350, 000. The bank offers you a 25-year mortgage with annual payments. The interest rate is 6%. What would be your annual payment so that you repay in 25 years the total mortgage loan that you obtain from the bank today?

Answer:

Timeline: From the bank's point of view:

0	1	2	3	...	25
$-350,000$	C	C	C	...	C

The NPV must be zero for the bank to agree to offer this mortgage. Hence, applying the annuity formula, we have:

$$NPV = -350000 + \frac{C}{0.06}\left[1 - \left(\frac{1}{1+0.06}\right)^{25}\right] \tag{16}$$

$$0 = -350000 + \frac{C}{0.06}\left[1 - \left(\frac{1}{1+0.06}\right)^{25}\right] \tag{17}$$

$$\text{Solving for } C \quad : \tag{18}$$

$$C = \frac{350000\,(0.06)}{\left[1 - \left(\frac{1}{1+0.06}\right)^{25}\right]} \tag{19}$$

$$= \frac{350000\,(0.06)}{[1 - (1.06)^{-25}]} \tag{20}$$

$$= \frac{21000.0}{0.76700137} \tag{21}$$

$$= 27379.351 \tag{22}$$

$$\approx \$27,379.35 \tag{23}$$

The payments per year must be $\$27,379.35$

(10pts) 7. Your parents would like to ensure that your little sister has enough funds to finance her university education. She is currently 6 years old and will need $150,000 in 12 years from now. The interest rate on savings accounts offered by their bank is 6% per year. How much money should your parents deposit in the savings account today so that your sister may have $150,000 in 12 years?

Answer:

Timeline:

0	1	2	3	...	12
?					150000

$$PV_0 = \frac{150000}{(1.06)^{12}} \tag{24}$$

$$= \$74545.405 \tag{25}$$