Carleton University School of Mathematics and Statistics

MATH1009F Fall 2019 TEST 1

Name: Student ID: /25

Non-programmable calculators are allowed. This test has 8 questions. **Please write all your solutions on this sheet. Make sure to write your name and student ID. Show your work.**

1. Use the definition of the logarithm to evaluate (without a calculator) the following logarithms: a) $\log_3 81$ (1 point) b) $\log_{\frac{1}{4}} 64$ (1 point)

2. Consider the function $f(x) = \begin{cases} x^2 - 2, & \text{if } x \leq -2, \\ x^2 + 4, & \text{if } -2 < x < 1, \\ 2x^3 + 3, & \text{if } x \geq 1. \end{cases}$

a) Find $\lim\limits_{x\to -2-} f(x)$, $\lim\limits_{x\to -2+} f(x)$, $\lim\limits_{x\to 1-} f(x)$ and $\lim\limits_{x\to 1+} f(x)$. (2 points)
b) Determine if $\lim\limits_{x\to -2} f(x)$ and $\lim\limits_{x\to 1} f(x)$ exist. (1 point)
c) Determine if $f(x)$ is continuous at $x = 1$. Justify. (1 point)

3. Evaluate the following limits. We only look at the final answer.
a) $\lim\limits_{x\to 3} \left(\dfrac{6}{x} - \dfrac{9}{\sqrt{4x-3}} \right)$ (1 point) b) $\lim\limits_{x\to -3} \dfrac{x^3 + 5x^2 + 6x}{x^2 - 2x - 15}$ (**Hint:** $\frac{0}{0}$) (1 point)
c) $\lim\limits_{x\to +\infty} \dfrac{5x^3 + 2x + 4}{2x^3 - 4x^2 + 5}$ (1 point) d) $\lim\limits_{x\to +\infty} \dfrac{x^2 + 5x + 8}{5x^3 + 2x^2 + 9x + 3}$ (1 point)

4. a) Use the properties of logarithms to expand the expressions $\ln\left(\dfrac{y^3}{x^2 z^6}\right)$.
(Your final answer should contain three natural logarithms). (1 point)

b) Use properties of the logarithms to write the expression $5\ln a - 3\ln b + 4\ln c - 2\ln d$ as a single logarithm. (1 point)

5. Solve for x the following equations

a) $4^{3x+6} \cdot 2^{2x-7} = 2^{-2x+3}$. Show your work. (2 points)
b) $\ln(2x + 7) = 8$. Show your work. (2 points)

6. Use intervals to describe the domain of the function $f(x) = 3\sqrt{16 - x^2}$.
(3 points)

7. The amount of \$2,000 is deposited in a bank that pays interest at the rate of 4 % per year compounded quarterly. How many years would it take for the investment to reach \$5000? (3 points)

8. Use the definition of the slope of the tangent line (and only that definition) to find the slope of the tangent line of the function $f(x) = 3x^2 + 2x + 1$ at $x = 2$. Show all your work and simplify your answer. **Only a solution using the definition of the derivative will be accepted.** (3 points)

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MATH1009F Fall 2019 Answer Sheet

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Please write your answers on the answer sheets. Do not forget to write your name and student ID on each answer sheet that you use. Please use the back of the sheet if you need more space.

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