

# MAT2377C - Assignment 4

## Solutions

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**Q1.** Assume that  $X$  is normal with mean 10 and variance 25. Find

- (a)  $P(X \leq 9)$
- (b)  $P(X > 11)$
- (c)  $P(9 < X \leq 11)$

**Solution to Q1:**

Use the normal table:

(a)

$$P(X \leq 9) = P\left(\frac{X - 10}{5} \leq \frac{9 - 10}{5}\right) = P(Z \leq -0.2) = 0.4207.$$

You may verify the answer by typing in R:

`pnorm(9,10,5)`

(b)

$$P(X > 11) = 1 - P(X \leq 11) = 1 - P\left(\frac{X - 10}{5} \leq \frac{11 - 10}{5}\right) = 1 - P(Z \leq 0.2) = 1 - 0.5793 = 0.4207.$$

You may verify the answer by typing in R:

`1-pnorm(11,10,5)`

(c)

$$P(9 < X \leq 11) = P(X \leq 11) - P(X \leq 9) = 0.5793 - 0.4207 = 0.1593$$

*Marking scheme for Q1:*

This question will not be marked.

**Q2.** (4 points) Assume that  $X$  is normal with mean 10 and standard deviation 3. Find the value  $x$  such that

- (a)  $P(X > x) = 0.5$
- (b)  $P(X > x) = 0.95$
- (c)  $P(x < X \leq 10) = 0.2$
- (d)  $P(-x < X - 10 \leq x) = 0.95$

**Solution to Q2:**

Using the normal table:

(a)  $0.5 = P(X > x)$ , thus

$$0.5 = P(X \leq x) = P((X - 10)/3 \leq (x - 10)/3) = P(Z < (x - 10)/3) = 0.5$$

Thus,  $(x - 10)/3 = 0 \Rightarrow x = 10$ .

You may verify the answer by typing in R:

`qnorm(0.5,10,3)`

(b)  $0.95 = P(X > x)$ , thus thus

$$0.05 = P(X \leq x) = P((X - 10)/3 \leq (x - 10)/3) = P(Z < (x - 10)/3) = 0.05$$

Thus,  $(x - 10)/3 = -1.645 \Rightarrow x = 5.065$ .

You may verify the answer by typing in R:

`qnorm(0.05,10,3)`

(c) Write

$$0.2 = P(x < X < 10) = P(X < 10) - P(X < x) = P(Z < (10 - 10)/3) - P(Z < (x - 10)/3).$$

We have  $P(Z < (10 - 10)/3) = P(Z < 0) = 0.5$ . This implies  $P(Z < (x - 10)/3) = 0.3$ . Thus,  $(x - 10)/3 = -0.52 \Rightarrow x = 8.44$ .

(d) Write

$$0.95 = P(-x < X - 10 < x) = P(X - 10 < x) - P(X - 10 < -x) = P(Z < x/3) - P(Z < -x/3).$$

By symmetry of the standard normal density

$$P(Z < -x/3) = P(Z > x/3) = 1 - P(Z < x/3).$$

Thus

$$0.95 = P(Z < x/3) - (1 - P(Z < x/3)) = 2P(Z < x/3) - 1$$

and  $P(Z < x/3) = (0.95 + 1)/2 = 0.975$ . Thus,  $x/3 = 1.96 \Rightarrow x = 5.88$ .

*Marking scheme for Q2:*

1 point for each correct answer. No partial marks. Total - 4 points.

**Q3.** (3 points) Time to reaction to a visual signal follows normal distribution with mean 0.5 seconds and standard deviation 0.035 seconds.

- (a) What is the probability that time to react exceeds 1 second?
- (b) What is the probability that time to react is between 0.4 and 0.5 seconds?
- (c) What is reaction time that is exceeded with probability of 0.9?

**Solution to Q3:**

I will write shortly  $\Phi(x)$  for  $P(Z < x)$ . We have  $X \sim N(0.5; (0.35)^2)$ .

- (a)  $P(X > 1) = 1 - \Phi[(1 - 0.5)/0.035] = 1 - \Phi(14.29) \approx 0$ .
- (b)  $P(0.4 < X < 0.5) = \Phi[(0.5 - 0.5)/0.035] - \Phi[(0.2 - 0.5)/0.035] = \Phi(0) - \Phi(-2.86) = 0.4979$ .
- (c) We want to find  $x$  such that  $0.90 = P(X > x) = 1 - \Phi[(x - 0.5)/0.035]$ . Thus,  $\Phi[(x - 0.5)/0.035] = 0.10 \Rightarrow (x - 0.5)/0.035 = -1.28 \Rightarrow x = 0.4552$ .

*Marking scheme for Q3:*

1 point for each correct answer. No partial marks. Total - 3 points.

**Q4.** Suppose that samples of size  $n = 25$  are selected at random from a normal population with mean 100 and standard deviation 10. What is the probability that sample mean falls in the interval

$$(\mu_{\bar{X}} - 1.8\sigma_{\bar{X}}, \mu_{\bar{X}} + 1.0\sigma_{\bar{X}})?$$

**Solution to Q4:**



- Q7.** (4 points) A random sample of size  $n_1 = 16$  is selected from a normal population with a mean of 75 and standard deviation of 8. A second random sample of size  $n_2 = 9$  is taken independently from another normal population with mean 70 and standard deviation of 12. Let  $\bar{X}_1$  and  $\bar{X}_2$  be the two sample means. Find
- (a) The probability that  $\bar{X}_1 - \bar{X}_2$  exceeds 4.
  - (b) The probability that  $3.5 < \bar{X}_1 - \bar{X}_2 < 5.5$ .

**Solution to Q7:**

$$\bar{X}_1 - \bar{X}_2 \sim N(75 - 70, 8^2/16 + 12^2/9) = N(5, 20).$$

(a)

$$P(\bar{X}_1 - \bar{X}_2 > 4) = P(Z > \frac{4 - 5}{\sqrt{20}}) = 1 - \Phi\left(\frac{4 - 5}{\sqrt{20}}\right) = 1 - \Phi(-0.22) = 0.5871.$$

(b)

$$\begin{aligned} P(3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5) &= \Phi\left(\frac{5.5 - 5}{\sqrt{20}}\right) - \Phi\left(\frac{3.5 - 5}{\sqrt{20}}\right) \\ &= \Phi(0.11) - \Phi(-0.34) \\ &= 0.5438 - 0.3669 = 0.1769 \end{aligned}$$

**Marking scheme for Q7:**

For each part: 1 point for the correct standardization, 1 point for the correct answer. Total - 4 points.