Assignment #1
Linear Programming (LP)
Formulation, Graphical Method and Excel Solver
Solutions and Marking Scheme

ADM2302 students are reminded that submitted assignments must be typed (i.e. **can NOT be hand written**), neat, readable, and well-organized. However, GRAPHS are ok to plot them by hand and SCAN/INCLUDE them within the Word document file as long as they are large, legible, and properly labeled and that their calculations are typed within the rest of the assignment. Assignment marks will be adjusted for sloppiness, poor grammar, spelling, for technical errors as well as if you submit a PDF file. (i.e. Do NOT submit a PDF file).

The assignment is to be submitted electronically as a **single Word Document file** via Brightspace by Sunday September 29th prior to 23:59. Front page of the Word document has to include title of the assignment, course code and section, student name and student number. Second page is *the individual statement of integrity that must be signed.*

Note: *Each student must provide an individual original submission of completed Assignment #1*. Please also note: Assignment #1 copies that are submitted jointly (i.e., by more than one author) will not be graded.
E-mail questions related to the assignment should be sent to the Teaching Assistant or posted on the Brightspace course website "Discussion page" (viewed by all).

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TOTAL: 60 points

General marking rules

Penalty for last submission:
No Penalty if submission is delayed by one hour or so.
10% deduction for less than 24 hours
25 % deduction for 1 day late
50 % deduction for 2 days late
70% deduction for 3 days late
100% deduction for 4 days late

- After inputting the final grade on the 1st page, if the statement of integrity is not available or not signed by a given student then deduct 6 points. Also ask the students to send you the statement of integrity on brightspace otherwise they will receive ZERO.
- On the front page, show the "original grade" - 6 (for missing statement of integrity) = "new grade".

- Don't penalize twice for an error that occurs at the start and it does affect the results that follows.
- Provide a brief explanation that would allow the students to understand were the error was committed and know what the right solution is.
- **Please provide me with the most common mistakes, so I can provide feedback to the students and go over the concepts in class.**

Problem 1 (18 points)

Dwight is an elementary school teacher who also raises pigs for supplemental income. He is trying to decide what to feed his pigs. He is considering using a combination of pig feeds available from local suppliers. He would like to feed the pigs at minimum cost while also making sure each pig receives an adequate supply of calories and vitamins. Dwight has formulated the following Linear Programming (LP) model:

Let A = pounds of Feed Type A in diet
B = pounds of Feed Type B in diet

Minimize Z =	$\$0.40A + \$0.80B$	(Cost)
subject to	$800A + 1{,}000B \geq 8{,}000$	(requirement of calories per day)
	$140A + 70B \geq 700$	(requirement of units of vitamins)
	$A \leq (1/3)(A + B)$	(constraint 3)
	$A \geq 0, B \geq 0.$	

Briefly explain or define each of these parts of the model:

a. The 0.80 in the objective function.
b. The product of the 0.40 and A in the objective function (e.g. 0.40A)
c. The 8,000 calories in the calories constraint.
d. The product of 1,000 and B in the calories constraint (e.g. 1,000B)
e. The 70 units in the vitamins constraint.
f. Constraint 3 was added since Feed Type A contains an ingredient that is toxic if consumed in too large a quantity. In managerial terms (e.g. verbally explain) what does constraint 3 mean?

Solve this LP problem using the graphical method:

g. Graph the constraints and identify the feasible region.
h. Determine the optimal solution(s) and the minimum cost (show your calculations). **Include "managerial statements" that communicate the results of the analyses (i.e. describe verbally the results).**

i. Determine the amount of slack for each of the constraint.

SOLUTIONS

a. The \$0.8 refers to the cost per pound for feed type B. (1 point)

b. This is the total cost of purchasing feed type A. (1 point)

c. "8000" calories refer to minimum requirement of calories per day (i.e. the pigs requires a least 8000 calories per day). (1 point)

d. The total amount of calories content from feed type B. (1 point)

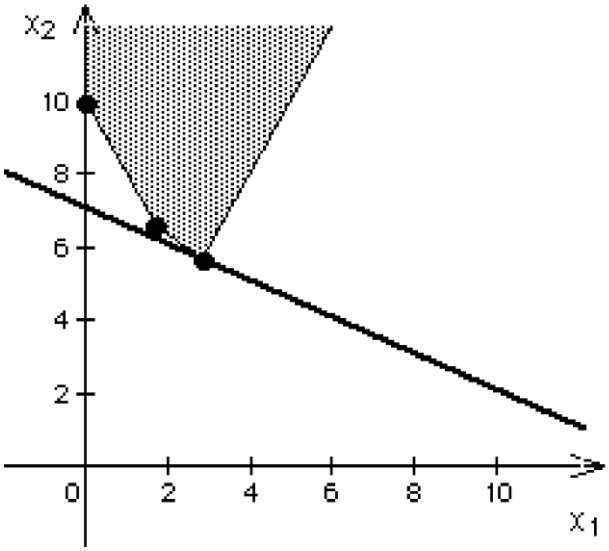
e. 70 is the vitamin content per pound of feed type B. (1 point)

f. No more than 1/3 of the diet (by weight) can consist of feed Type A. (1 point)

g. ***(6 points)***

4 points: treat the constraint correctly.
Deduct 2 points for wrong plotting of constraint 3 (Make sure that its slope is correct)
Note that x_1 is equivalent to A and x_2 is equivalent to B in the graph below

2 points find the feasible region



h. **(4 points)**

1 point for "managerial statement", e.g., It is recommended that Dwight feed his pigs 2.85715 pounds of Feed Type A and 5.71428 pounds of feed type B ; (2) for a total minimum cost of $5.72.

3 points: solve correctly for the optimal solution below using the algebraic method (two equations with two unknown) and not by eye balling
Please note, that in this question I did ask to use the Isocost line method. (If not then deduct 2 points)

Optimal Solution: $(A, B) = (x_1, x_2) = (2.86, 5.71)$ and $C = \$5.72$.

i. 2 points (Deduct 1 point per mistake)

The amount of slack/surplus for each constraint is:

Calories per day = [800(2.85715) + 1000(5.71428)] – 8000 = 8000 – 8000 = 0
Vitamins = [140(2.85715) + 70(5.71428)] – 700 = 800 – 700 = 100 units of vitamins
Constraint 3 = 0

Problem 2 (10 points)

a. Solve the following linear programming model by using the graphical method: graph the constraints and identify the feasible region then determine the optimal solution (s) (show your work).

Minimize $Z = 3x_1 + 7x_2$

Subject to

$9x_1 + 3x_2 \geq 36$ **(constraint 1)**
$4x_1 + 5x_2 \geq 40$ **(constraint 2)**
$x_1 - x_2 \leq 0$ **(constraint 3)**
$2x_1 \leq 13$ **(constraint 4)**
$x_1, x_2 \geq 0$

b. Are any constraints binding? If so, which one (s)?

a. (8 points)

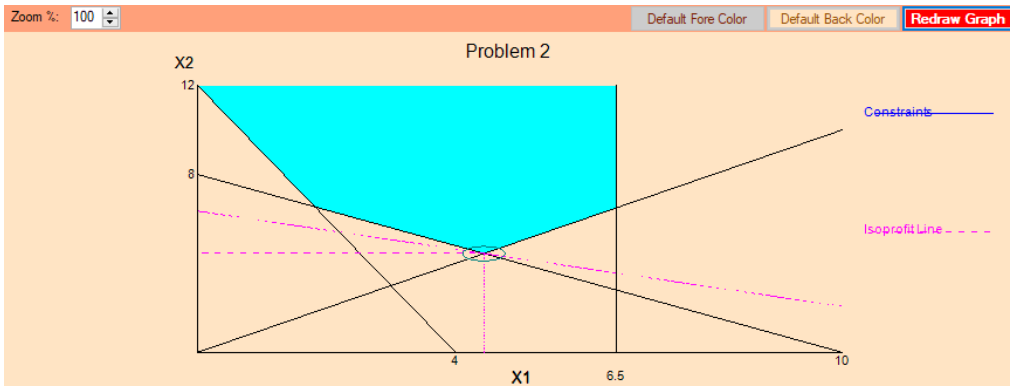
4 points: treat the constraints correctly.

Please make sure that constraint 3 ($x_1 - x_2 \leq 0$) has the correct slope. Thus the points on that equation line must be x1=x2. For example, (0,0); (2,2), (6.5, 6.5)...**Otherwise deduct: 2 points**

2 points find the feasible region

2 points finding the optimal solution and Z

$(x_1, x_2) = (4.44, 4.44)$ and $Z = 44.44$.



b. 2 points: constraint 2 and constraint 3 are binding as the optimal solution lies on the intersection of those two constraints (i.e. the slack is Zero for both constraints).

Problem 3 (14 points)

a. Solve the following linear programming model by using the graphical method: graph the constraints and identify the feasible region. Using the corner points method, determine the optimal solution (s). (show your work).

Maximize $Z = 6.5x_1 + 10x_2$

Subject to

$x_1 + x_2 \leq 15$

$2x_1 + 4x_2 \leq 40$

$x_1 \geq 8$

$x_1, x_2 \geq 0$

b. If the constraint $x_1 \geq 8$ is changed to $x_1 \leq 8$, what effect does this have on the optimal solution? Are any constraints redundant? If so, which one (s)?

Solutions:

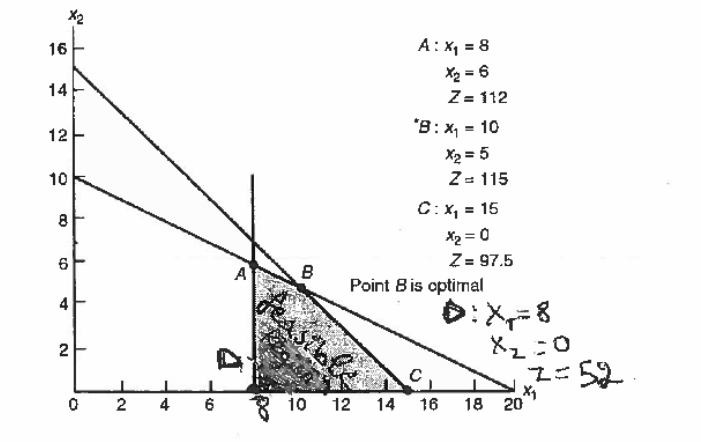
a. (10 points)

3 points: treat the constraints correctly.

2 points find the feasible region.

5 points: to solve for the optimal solution point B (x1= 10, x2=5 and Z = 115).

Please note that in this question I did ask to use the corner point method (deduct 2 points for not using it).



b. 4 points

2 points: It changes the optimal solution to Point A (x1 = 8, x2 = 6, Z = 112)

2 points: constraint $x_1 + x_2 \leq 15$ is redundant as it is no longer part of the solution space boundary (i.e. not part of the boundary of the feasible region).

Problem 4 (18 points)

A farmer must decide what crops to grow on a 300-hectare tract of land. He can grow oats, wheat, or barley, which yield 50, 100 and 80 kg/hectare (respectively) and sell for $1.00, $0.80, and $0.60 per kg (respectively). Production costs (fertilizer, labor, etc.) are $40, $50, and $40 per hectare for growing oats, wheat and barley, respectively. Government regulations restrict the farmer to a maximum of 150 hectares of wheat and his crop rotation schedule requires that he plants at least 50 hectares in oats and 50 hectares in barley. Because of his storage arrangements, the farmer wants the number of hectares of oats to be equal to or less than half the number of hectares of barley.

a. Formulate algebraically the linear programming model of this problem that will maximize the farmer profit (i.e. revenue – cost) and help him/her decides what crops to grow on his/her land (i.e. define the decision variables, objective function, constraints).
b. Formulate this same linear programming problem on a spreadsheet and SOLVE using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). **Include "managerial statements" that communicate the results of the analyses.**

Solution (Check the Excel file: Problem 4.xlsx)

a. Algebraic formulation **(10 points)**

2 points: for decision variables and their definition
2 points: objective function
6 points: Constraints (for each mistake or missing constraint deduct 1 point)

Decision Variables:
O = hectares used for oats
W = hectares used for wheat
B = hectares used for barley
Objective Function:
Maximum Z = 50*(1.00 - 0.80)*O + 100*(0.80 - 0.50)*W + 80*(0.60 - 0.50)*B
Subject to:

O + W + B <= 300	(total hectares available)
W <= 150	(Maximum wheat)
O >= 50	(Minimum oats)
B >= 50	(Minimum barley)
O - (1/2)B <= 0	(oats-to-barley requirement)

b. (**8 points)**

Solution using Excel Solver: 6 points: for setting the spreadsheet in "Excel Solver" and Solving.

- I asked the student to print the excel spreadsheet and the answer report. If one is provided but not the other just deduct 2 points
- If the solution is wrong due to the errors ONLY committed in part a, just deduct 2 points.
- If the student writes down the non negativity constraint in the EXCEL spreadsheet formulation **then deduct 2 points** Non negativity constraints is to be added by clicking the box under excel solver.

2 points for the "managerial statement: the farmer should grow 50 hectares of oats, 150 hectares of wheat, 100 hectares of barely for a Maximum profit of $5,800.