A sample Mid term test for MAT2375, The students were tested in winter 2019.

Part A: Detailed answer is required

1. Let $X_1, \ldots, X_n$ be a sequence of i.i.d. observations from an exponential distribution

$$f(x;\theta) = \theta \exp(-\theta x), \quad x > 0.$$

where $\theta > 0$ is an unknown parameter. Find the maximum likelihood estimate for θ.

Answer is

$$\frac{1}{\bar{X}}$$

2. Consider the model

$$Y_i = 2 + \beta x_i + \epsilon_i, \ i = 1, 2, \ldots, n$$

where $\epsilon_1, \ldots, \epsilon_n$ be a sequence of i.i.d. observations from $N(0, \sigma^2)$. and $x_1, \ldots, x_n$ are given constants.

(i) Find MLE for β and call it $\hat{\beta}$..

Answer is

$$\hat{\beta} = \frac{\sum_{i=1}^n (Y_i - 2)x_i}{\sum_{i=1}^n x_i^2}.$$

(ii) For a sample of size $n = 7$ let $(x_1, \ldots, x_7) = (0, 0, 1, 2, 3, 5, 6)$ and $(y_1, \ldots, y_7) = (2, 4, 5, 8, 8, 10, 13)$. Calculate $\hat{\beta}$.

Answer is: Since

$$\sum_{i=1}^n x_i^2 = 75, \sum_{i=1}^n (Y_i - 2)x_i = 139$$

we get

$$\hat{\beta} = \frac{139}{75} = 1.853333.$$

Part B: Multiple choice questions. No penalty for wrong answers

1. Let $X_1, \ldots, X_5$ be a sequence of i.i.d. observations from

$$f(x) = 2x, \quad 0 \le x \le 1.$$

Let $Y_1 < Y_2 < \cdots < Y_5$ be their ordered Statistics. Then $E(Y_4)$ is

A: 0.254 B: 0.353 C: 0.808 D: 0.765 E: 0.808.

The right answer is C.

2. Let $X_1, \ldots, X_{100}$ be a sequence of i.i.d. observations from the same distribution

$$f(x) = 2x, \quad 0 \le x \le 1.$$

Then $Var(\sum_{i=1}^{n} X_i)$ is

A: 1.978 B: 4.096 C: 9.904 D: 5.556 E: 7.896.

The right answer is D since $Var(\sum_{i=1}^{100} X_i) = 100Var(X_1)$.

3. Let $X_1, \ldots, X_n$ be a sequence of random variables from a uniform distribution on $[\theta, 2\theta]$ for $\theta > 0$. Using the method of moments the estimator for the unknown parameter θ is

A: $\bar{X}$ B: $2\bar{X}$ C: $3\bar{X}/4$ D: $2\bar{X}/3$ E: $\bar{X}/3$.

The right answer is D since $E(X) = 3\theta/2$.

4. Let $X_1, \ldots, X_n$ be a sequence of i.i.d. observations from an exponential distribution

$$f(x;\theta) = \frac{1}{6\theta^4} x^3 \exp(-x/\theta), 0 \leq x.$$

where $\theta > 0$ is an unknown parameter. Find the maximum likelihood estimate for θ.

A: $\bar{X}/3$ B: $\bar{X}/4$ C: $3\bar{X}$ D: $4\bar{X}$ E: $2\bar{X}$.

The right answer is B.

5. A manufacturing company introduces a new chemical (chemical B) to have a more rapid production line. In a sample of 200 manufactured items using material A the average time of production was 10 minutes with sample variance of $S_A^2 = 7$. The same experiment with chemical B in a 200 random sample results the average time of 12 minutes with sample variance of $S_B^2 = 9$. Find a 95% confidence interval for $\mu_A - \mu_B$ given that $\sigma_A^2 = \sigma_B^2$

A: 2 ± 0.55 B: -2 ± 0.55 C) 2 ± 0.66 D) -2 ± 0.66 D: -2 ± 0.88

Answer is B. Notice that you need to use

$$\bar{x} - \bar{y} \pm z_{\alpha/2} s_p \sqrt{1/n + 1/m}.$$

In here the degrees of freedom (198) is large and both t values and z values will be the same and $s_p^2 = 8$.

6. The average height of $n = 45$ randomly selected female olympic athletes in gymnastic is 67 inches and their variance is 38 inches. A 95% confidence interval for the average height of the female Olympics athletes in gymnastic is

A: 67 ± 1.8 B: 67 ± 5.76 C: 67 ± 1.10 D: 67 ± 9.32 E) 67 ± 1.51

Answer is E. Simply use

$$\bar{x} \pm t_{\alpha/2} s/\sqrt{n}.$$

7. Based on a random sample of $n = 15$ males, a 99% confidence interval for average life expectancy is $[75.49, 80.51]$. Assuming life expectancy follows a normal distribution determine the sample standard deviation.

A: 3.24 B: 1.51 C: 2.43 D: 2.51 E) 3.24.

Answer. The difference between upper and lower bound is

$$2t_{0.005}s/\sqrt{n} = 80.51 - 75.49 - 2.51.$$

Replace $n = 15$ and $t = 2.997$ to find $s = 3.24$.

8. After a TV ad campaign in order to increase the milk consumption for adult Canadian females in a random sample of size $n = 25$ females it was noticed that the average consumption of milk per day was 756 grams and its standard deviation was 35 grams. Assuming normality find a 95% confidence interval for the average milk consumption of an adult female.

A: 756 ± 2.15 B: 756 ± 14.45 C) 756 ± 15.45 D) 756 ± 11.76 D: 756 ± 12.76

Answer is B. Use

$$\bar{x} \pm t_{0.025}s/\sqrt{n} = 756 \pm 14.45.$$