CARLETON UNIVERSITY

FINAL EXAMINATION
April 2016

DURATION: 3 HOURS **SCANTRON FORMS REQUIRED**

Department Name and Course Number: School of Mathematics and Statistics, MATH 1005 A, B, C, D, E, F
Course Instructor(s): Dr. S. Melkonian (Section A), Mr. R.J. Cova (Sections B and D), Mr. R.C. Hayman (Section C), Dr. V. Nijimbere (Section E), Mr. M. Blenkinsop (Section F).

1. If y is the solution of the initial-value problem $\dfrac{dy}{dx} = \dfrac{3x^2}{2y}$, $y(0) = 1$, then $y(1) =$

   (a) $-\sqrt{2}$ (b) $\sqrt{2}$ (c) 0 (d) 2 (e) None of these

2. The orthogonal trajectories of the general solution of the equation $y' = \dfrac{y}{x}$ satisfy the equation

   (a) $y' = \dfrac{x}{y}$ (b) $y' = -\dfrac{y}{x}$ (c) $y' = \dfrac{y}{x}$ (d) $y' = -\dfrac{x}{y}$ (e) None of these

3. If y is the solution of the initial-value problem $xy' - 2y = x^3$, $y(1) = 0$, then $y(2) =$

   (a) 0 (b) 1 (c) 2 (d) 4 (e) None of these

4. The general solution of the differential equation $x^2\dfrac{dy}{dx} = xy - y^2$ is $y =$

   (a) $\dfrac{1}{c - \ln|x|}$ (b) $\dfrac{x}{c - \ln|x|}$ (c) $\dfrac{x}{\ln|x| - c}$ (d) $\dfrac{1}{\ln|x| - c}$ (e) None of these

5. The general solution of the equation $(3x^2y^2 + y^3 + 2) + (2x^3y + 3xy^2 - 3)\dfrac{dy}{dx} = 0$ is

   (a) $x^3y^2 + xy^3 + 2x = c$ (b) $x^3y^2 + xy^3 + 2x + 3y = c$ (c) $x^3y^2 + xy^3 + 2x - 3y = c$
   (d) $x^3y^2 + xy^3 - 3y = c$ (e) None of these

6. An integrating factor which makes the equation $xe^{x^2} + xy^2 + y\dfrac{dy}{dx} = 0$ exact is $I(x) =$

   (a) e^{x^2} (b) e^{-x^2} (c) $2x$ (d) $-2x$ (e) None of these

7. The general solution of the equation $y'' - 4y' + 4y = 0$ is $y =$

   (a) $c_1e^{2x} + c_2e^{2x}$ (b) $e^{2x}(c_1 + c_2x)$ (c) $x^2(c_1 + c_2\ln|x|)$ (d) $e^{2x}(c_1 + c_2\ln|x|)$
   (e) None of these

8. The solution of the initial-value problem $y'' + 4y = 0$, $y(0) = 0$, $y'(0) = 6$ is $y =$

   (a) $3\cos(2x)$ (b) $\frac{3}{2}\big(e^{2x} - e^{-2x}\big)$ (c) $6\sin(2x)$ (d) $3\sin(2x)$ (e) None of these

9. The general solution of the equation $x^2y'' - 3xy' + 3y = 0$ is $y =$

   (a) $c_1x + c_2x^3$ (b) $c_1e^x + c_2e^{3x}$ (c) $e^{\frac{3}{2}x}\big[c_1\cos(\frac{\sqrt{3}}{2}x) + c_2\sin(\frac{\sqrt{3}}{2}x)\big]$
   (d) $|x|^{\frac{3}{2}}\big[c_1\cos(\frac{\sqrt{3}}{2}\ln|x|) + c_2\sin(\frac{\sqrt{3}}{2}\ln|x|)\big]$ (e) None of these

10. The general solution of the equation $y'' - y = 6e^{2x}$ is $y =$

    (a) $2e^{2x} + c_1\cos(x) + c_2\sin(x)$ (b) $2e^{2x} + c_1e^x + c_2e^{-x}$ (c) $2xe^{2x} + c_1e^x + c_2e^{-x}$
    (d) $2x^2e^{2x} + c_1e^x + c_2e^{-x}$ (e) None of these

11. The general solution of the system $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, is

    (a) $c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (b) $c_1e^t\begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} + c_2e^t\begin{pmatrix} \sin(t) \\ -\cos(t) \end{pmatrix}$
    (c) $c_1e^t\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2e^t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (d) $c_1e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (e) None of these

12. The sum of the series $\displaystyle\sum_{n=0}^{\infty} 2\cdot 3^{1+n}4^{1-n}$ is

    (a) 6 (b) 8 (c) 72 (d) 96 (e) None of these

13. The series $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n n^2}{\sqrt{n^4+1}}$

    (a) converges (b) diverges

14. The series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}}$

    (a) converges (b) diverges

15. The series $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^2-1}$

    (a) converges (b) diverges

16. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

    (a) converges absolutely (b) converges conditionally (c) diverges

17. The series $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^2+1}$

    (a) converges absolutely (b) converges conditionally (c) diverges

18. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

    (a) converges absolutely (b) converges conditionally (c) diverges

19. The radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-1)^n}{3^n}$ is $R =$

    (a) $\frac{1}{3}$ (b) 1 (c) ∞ (d) 3 (e) None of these

20. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$ is $I =$

    (a) $[0,2]$ (b) $(0,2)$ (c) $[0,2)$ (d) $(0,2]$ (e) None of these

21. The Maclaurin series (Taylor series about 0) of $f(x) = e^{2x}$ is

    (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}x^n$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}x^{2n}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}x^n$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}(x+2)^n$ (e) None of these

22. The coefficient of $(x-1)^3$ in the Taylor series of $f(x) = \ln(x)$ about (centred at) 1 is

    (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 2 (d) $-\frac{1}{3}$ (e) None of these

23. The coefficient of x^3 in the Maclaurin series of $f(x) = \dfrac{1}{\sqrt{1+x}}$ is

    (a) $\frac{5}{8}$ (b) $-\frac{5}{8}$ (c) $\frac{5}{16}$ (d) $-\frac{5}{16}$ (e) None of these

24. Let $f(x) = \left\{ \begin{array}{l} 1,\ 0 \le x < 1 \\ 3,\ 1 \le x \le 3 \end{array} \right\}$ for $0 \le x \le 3$, and $f(x+3) = f(x)$ for all x. At $x = 5$, the Fourier series of f converges to

    (a) 1 (b) 2 (c) 3 (d) 0 (e) None of these

25. Let $f(x) = x$ for $0 \le x \le 1$. The Fourier cosine series of f on $[0, 1]$ is

(a) $1 + \displaystyle\sum_{n=1}^{\infty} \frac{2[(-1)^n - 1]}{n^2\pi^2} \cos(n\pi x)$ (b) $\frac{1}{2} + \displaystyle\sum_{n=1}^{\infty} \frac{2[(-1)^n - 1]}{n^2\pi^2} \cos(n\pi x)$

(c) $\frac{1}{2} + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2\pi^2} \cos(n\pi x)$ (d) $\frac{1}{4} + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2\pi^2} \cos(n\pi x)$ (e) None of these

Answers

1. b
2. d
3. d
4. c
5. c
6. a
7. b
8. d
9. a
10. b
11. a
12. d
13. b
14. b
15. a
16. a
17. c
18. b
19. d
20. c
21. c
22. a
23. d
24. c
25. b