MATH 1009-E TEST 1: Solutions September 2019

PART A: MULTIPLE CHOICE QUESTIONS.

A1. The domain of the function $f(x) = \sqrt{9-x}$ is

(a) $[3, \infty)$, or, equivalently, $\{x \geq 3\}$.

(b) $(-\infty, 3]$, or, equivalently, $\{x \leq 3\}$.

(c) $[9, \infty)$, or, equivalently, $\{x \geq 9\}$.

(d) $(-\infty, 9]$, or, equivalently, $\{x \leq 9\}$.

(e) $\mathbb{R}$, all real numbers.

Answer: (d)

A2. The domain of the function $f(x) = \log_2(x-1)$ is

(a) $[1, \infty)$, or, equivalently, $\{x \geq 1\}$.

(b) $(1, \infty)$, or, equivalently, $\{x > 1\}$.

(c) $\mathbb{R}$, or, equivalently, $(-\infty, \infty)$.

(d) $(-1, \infty)$, or, equivalently, $\{x > -1\}$.

(e) $[-1, \infty)$, or, equivalently, $\{x \geq -1\}$.

Answer: (b)

A3. The expression $(7^{-2/3} \cdot 7^2)^{3/4}$ evaluates to

(a) -7. **(b)** 7. **(c)** $\frac{1}{7}$. **(d)** $-\frac{1}{7}$. **(e)** None of the above.

Answer: (b)

A4. The expression $\dfrac{e^{1.2} \cdot (e^{0.1})^{-2}}{e^{-5}}$ simplifies to

(a) $e^{4.3}$. (b) $e^{-5.7}$. (c) e^4. (d) e^5. (e) e^6.

Answer: (e)

A5. The expression $(9x^4y^6)^{1/2}$ simplifies to

(a) $3x^2y^3$. (b) $3x^2y^4$. (c) $\dfrac{9x^4y^6}{2}$. (d) $9x^2y^3$. (e) None of the above.

Answer: (a)

A6. The expression $\log_2 \dfrac{1}{2}$ evaluates to

(a) -2. (b) 2. (c) -1. (d) 1. (e) None of the above.

Answer: (c)

A7. Write the expression $(\ln 2 + \frac{1}{3}\ln x - 2\ln y)$ as a single logarithms:

(a) $\ln\left(\dfrac{x}{3y}\right)$. (b) $\log_2\left(\dfrac{2x^{1/3}}{y^2}\right)$. (c) $\ln\left(2 + \dfrac{x}{3} - 2y\right)$. (d) $\ln\left(\dfrac{2x^{1/3}}{y^2}\right)$.

(e) None of the above.

Answer: (d)

PART B: LONG STYLE QUESTIONS.

[7 marks] B1. The weekly demand and supply equations for a company are given by $p = 68 - x^2$ and $p = 14 + \frac{1}{2}x^2$, respectively, where p is the price measured in dollars and x is measured in units **of a thousand**.

[2] (a) For the demand equation $p = 68 - x^2$, determine the quantity demanded, when the price is set at 4 dollars.

$$4 = 68 - x^2, \quad x^2 = 64, \quad x = 8.$$

We reject the negative root $x = -8$, since only positive values of the quantity demanded are meaningful.

[1] (b) For the supply equation $p = 14 + \frac{1}{2}x^2$, determine the price at which the supplier will make 2 thousand units available in the market.

$$p = 14 + \frac{1}{2} \cdot 2^2 = 14 + 2 = 16.$$

[4] (c) Find the equilibrium quantity and price.

At the equilibrium point the supply is equal to the demand, and therefore

$$68 - x^2 = 14 + \frac{1}{2}x^2.$$

Solving this equation for x yields $\frac{3}{2}x^2 = 68 - 14 = 54, \quad x^2 = 36, \quad x = 6$. (We reject the negative root $x = -6$, since only positive values of x are meaningful.) Thus, the equilibrium quantity is 6 thousand units, and the corresponding price is $p = 68 - 6^2 = 32$ dollars per unit.

[12 marks] B2. Solve each of the following equations for x.

[4] (a) $e^{x-2} = 3$ **[4] (b)** $\log_3(x^2 - 8) = 0$ **[4] (c)** $\ln x - \ln 2 + \ln 4 = 3$

Solution:

NOTE: There is more than one way of solving each of the equations.

(a) Take the natural logarithm of both sides of the equation and use the laws of logarithms:

$$\ln(e^{x-2}) = \ln 3 \quad \Rightarrow \quad x - 2 = \ln 3 \quad \Rightarrow \quad x = \ln 3 + 2.$$

(b) Since the logarithm equals zero, the argument must be equal to 1:

$$x^2 - 8 = 1 \quad \Rightarrow x^2 = 9 \quad \Rightarrow x = \pm 3.$$

(c) Simplify the LHS of the equation and then exponentiate both sides of the relation:

$$\ln\left(\frac{x \cdot 4}{2}\right) = 3 \quad \Rightarrow \quad \ln(2x) = 3 \quad \Rightarrow \quad e^{\ln(2x)} = e^3 \quad \Rightarrow \quad 2x = e^3 \quad \Rightarrow \quad x = e^3/2.$$

[**7 marks**] **B3**. The amount of $20,000 is deposited in a bank that pays interest at the rate of 6% per year compounded **semiannually**. Using the compound interest formula

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt},$$

answer the following questions.

[1] **(a)** What is the accumulated amount on deposit in 5 years? (two decimals)

$$A(5) = 20,000\left(1 + \frac{0.06}{2}\right)^{2\cdot 5} = 20,000(1.03)^{10} = 26,878.33 \text{ (dollars)}.$$

[1] **(b)** What is the interest earned in 5 years? (two decimals)

$$\text{Interest } I(5) = A(5) - P = 26,878.3 - 20,000 = 6,878.33 \text{ (dollars)}.$$

[5] **(c)** How long will it take to double the investment? (one decimal)

$$40,000 = 20,000\left(1 + \frac{0.06}{2}\right)^{2\cdot t} \rightarrow 2 = (1.03)^{2t} \rightarrow \ln 2 = 2t\ln(1.03) \rightarrow t = \frac{\ln 2}{2\ln 1.03} = 11.7 \text{ (years)}.$$