Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2011	3
Instructors		**Course Examiners**
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Special Instructions

▷ **Only Sharp EL 531 or Casio FX 300 MS calculators are allowed.**

MARKS

[11] **1.** (a) Sketch the graph of the function $f(x) = |x^2 - 2x|$ starting from the graph of the standard parabola and using appropriate transformations.

(b) Suppose $f(x) = x^3 - 1$ and and $g(x) = \sqrt[3]{1+x}$. Find $f \circ g$, $g \circ f$, and $f \circ f$.

(c) Find the inverse of the function $f(x) = 2 - e^x$. Determine the domain and range of f and f^{-1}.

[8] **2.** Evaluate the limits:

(a) $\lim\limits_{x \to -4} \dfrac{\sqrt{x^2+9} - 5}{x+4}$ (b) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$

Do not use l'Hôpital's rule.

[11] **3.** (a) Consider the function $f(x) = \dfrac{|x-1|(x+3)}{x^2-1}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

(b) Find parameters a and b such that the function

$$f(x) = \begin{cases} \dfrac{|x+1|}{x+1}, & \text{if } x < -1 \\ ax, & \text{if } -1 \le x < 1 \\ bx + 2, & \text{if } x \ge 1 \end{cases}$$

will be continuous at every point.

[16] **4.** Find derivatives of the functions (you don't have to simplify the answers):

(a) $f(x) = \dfrac{(\sqrt{x} - 2x)^2 - 3x^2}{x}$;

(b) $f(x) = e^2 + e^{x^2} \ln(1 - 5x)$;

(c) $f(x) = \dfrac{\sin^3 x}{\cos(2x) + \tan(x)}$;

(d) $f(x) = \arctan\left(\sqrt{1 + \ln(3x - 1)}\right)$;

(e) $f(x) = (1 + \sin x)^{\tan(x)}$ (use logarithmic differentiation).

[12] **5.** Given the function $f(x) = \sqrt{36 - x}$,

(a) Use appropriate differentiation rules to find the derivative of the function.

(b) Derive the same result for $f'(x)$ using the definition of the derivative as the limit of the difference quotient.

(c) Find the linear approximation of $f(x)$ at $a = 0$.

(d) Use this linear approximation to approximate $\sqrt{32}$.

[17] **6.** (a) Verify that the point (2,1) belongs to the curve defined implicitly by the equation $x^2 + 2y^2 + 2 = x^3y^3$, and find an equation of the tangent line to the curve at this point.

(b) Two cars are moving away from the intersection of two orthogonal streets at the speeds $v_1 = 15$ m/s going east, and $v_2 = 20$ m/s going north. How fast is the distance between the cars increasing at the instant when the first car is at the distance $x = 30$ m and the second car is at the distance $y = 40$ m from the intersection?

(c) Use l'Hôpital's rule to evaluate $\lim\limits_{x \to 0} \dfrac{\sin^2(5x)}{1 - \cos(x)}$.

[6] **7.** Let $f(x) = \sqrt{1+4x}$.

(a) Find the slope m of the secant line joining the points $(0, f(0))$ and $(2, f(2))$.

(b) Find all point(s) $x = c$ (if any) on the interval $[0, 2]$ such that $f'(c) = m$.

[5] **8.** The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the printed material on the poster is a rectangle with an area of 384 cm^2, find the dimensions of the poster with the the smallest area.

[14] **9.** Given the function $f(x) = \dfrac{x^2}{x^2+9}$,

(a) Find the domain and check for symmetry. Find all horizontal and vertical asymptotes (if any).

(b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).

(d) Sketch the graph of the function.

[5] **Bonus Question**

Suppose f and g are both concave upward on $(-\infty, \infty)$. What condition on f is sufficient to ensure that the composite function $h(x) = f(g(x))$ is also concave upward?