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ELEC 2607 **Assignment 1** 20 Marks

1. Using the method of exhaustive proof show whether $\overline{a+b+c}$ is equal to $\overline{(a+b)} \cdot \bar{c}$. 2 marks

$\overline{a+b+c} = \bar{a} \cdot \bar{b} \cdot \bar{c}$ (DeM) => Dual DeMorgan's Law

⇓

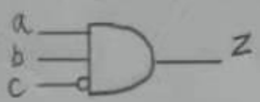
$(\bar{a} \cdot \bar{b}) \cdot \bar{c} = (\overline{a+b}) \cdot \bar{c}$ (DeM) => DeMorgan's Law

$= \overline{(a+b)} \cdot \bar{c}$

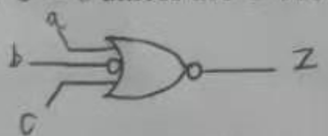
$\therefore \overline{a+b+c} = \overline{(a+b)} \cdot \bar{c}$

2. Draw the simplest logic gate or circuit for the following descriptions. The inputs are a, b, c. The output is z. 4 marks

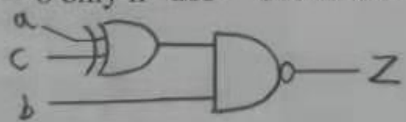
(a) $z = 1$ only if $abc = 110$



(b) $z = 0$ unless $abc = 010$



(c) $z = 0$ only if "$abc = 110$ or $abc = 011$"



3. Simplify the following 4 marks

(a) $(X + Y + \bar{Y}Z)(Y + X)(Y + \bar{X})$ to 1 literal

$= (X+Y+\bar{Y}Z)(X \cdot \bar{X}+Y)$ ···· (D2)
$= (X+Y+\bar{Y}Z)(0+Y)$ ···· (N2)
$= (X+Y+\bar{Y}Z) \cdot Y$ ···· (B1)
$= X \cdot Y + Y \cdot Y + Y \cdot \bar{Y} \cdot Z$ ···· (D1)
$= X \cdot Y + Y \cdot Y + 0 \cdot Z$ ···· (N2)
$= X \cdot Y + Y \cdot Y + 0$ ···· (B4)
$= X \cdot Y + Y + 0$ ···· (I2)
$= X \cdot Y + 1 \cdot Y + 0$ ···· (B2)
$= Y \cdot (X+1) + 0$ ···· (D1)
$= Y \cdot 1 + 0$ ···· (B3)
$= Y + 0$ ···· (B2) $= Y$ ···· (B1)

(b) $(BCD + C)CD$ to 2 literals

$(BCD+C)CD$
$= B \cdot C \cdot C \cdot D \cdot D + C \cdot C \cdot D$ ···· (D1)
$= B \cdot C \cdot D + C \cdot D$ ···· (I2)
$= B \cdot C \cdot D + 1 \cdot C \cdot D$ ···· (B2)
$= (B+1) \cdot C \cdot D$ ···· (D1)
$= 1 \cdot C \cdot D$ ···· (B3) $= C \cdot D$ ···· (B2)

(c) $A\bar{B} + AC + \bar{C}B$ to 3 literals

$= A\cdot\bar{B} + A\cdot C\cdot 1 + \bar{C}\cdot B$ ..... (B2)

$= A\cdot\bar{B} + A\cdot C\cdot(B+\bar{B}) + \bar{C}\cdot B$ ... (N1)

$= A\cdot\bar{B} + A\cdot C\cdot\bar{B} + A\cdot C\cdot B + \bar{C}\cdot B$ .... (D1)

$= \bar{B}\cdot(A + A\cdot C) + B\cdot(A\cdot C + \bar{C})$ .... (D1)

$= \bar{B}\cdot A + B\cdot(A\cdot C + \bar{C})$ ..... (S1)

$= \bar{B}\cdot A + B\cdot(\bar{C} + A)$ ..... (Ab1)

$= \bar{B}\cdot A + B\cdot A + B\cdot\bar{C}$ ..... (D1)

$= A\cdot(\bar{B} + B) + B\cdot\bar{C}$ ..... (D1)

$= A\cdot 1 + B\cdot\bar{C}$ ..... (N1)

$= A + B\cdot\bar{C}$ ..... (B2)

4. Prove algebraically that $(a+b)(b+c)(c+a) = ab + bc + ca$ 2 mark

$(a+b)(b+c)(c+a)$

$= (a\cdot c + b)(c+a)$ ..... (D2)

$= a\cdot c\cdot(c+a) + b\cdot(c+a)$ ..... (D1)

$= a\cdot c\cdot c + a\cdot a\cdot c + b\cdot c + a\cdot b$ ..... (D1)

$= a\cdot c + a\cdot c + b\cdot c + a\cdot b$ ..... (I2)

$= a\cdot c + b\cdot c + a\cdot b$ ..... (I1) $= ab + bc + ca$

$\therefore (a+b)(b+c)(c+a) = ab + bc + ca$.

5. Use duality on $\bar{a}\bar{b} + ab = (a+\bar{b})(\bar{a}+b)$ to find an alternate expression for $\bar{a}b + a\bar{b}$. 1 mark

$(\bar{a}+\bar{b})(a+b) = a\cdot\bar{b} + \bar{a}\cdot b$

6. Convert the following expressions to a form where the bar is over single variables only. 4 marks

(a) $\overline{a(b\bar{c} + de) + \overline{(d+a)}cg}$

$= \overline{a(b\bar{c}+de)} \cdot \overline{\overline{(d+a)}cg}$ ... (DeM)

$= [\bar{a} + \overline{(b\bar{c}+de)}] \cdot [(d+a) + \overline{cg}]$ .... (DeM)

$= [\bar{a} + (\overline{b\bar{c}} \cdot \overline{de})] \cdot [(d+a) + (\bar{c}+\bar{g})]$ ... (DeM)

$= \{\bar{a} + [(\bar{b}+c)\cdot(\bar{d}+\bar{e})]\} \cdot [(d+a) + (\bar{c}+\bar{g})]$ ... (DeM)

$= \{\bar{a} + [(\bar{b}+c)\cdot(\bar{d}+\bar{e})]\} \cdot (a + d + \bar{c} + \bar{g})$.

(b) $\overline{a(b\bar{c} + \overline{\bar{e}d}) + \overline{\overline{(d+ab)}\,\overline{(cg)}}} + a\overline{dcg}$

$= \overline{a[b\bar{c} + (e+\bar{d})]} \cdot \overline{[\overline{(d+ab)}\cdot\overline{(cg)}]} + a(\bar{d}+\bar{c}+\bar{g})$ ..... (DeM)

$= \bar{a} + \overline{[b\bar{c}+(e+\bar{d})]} \cdot \{[\bar{d}\cdot\overline{(ab)}]\cdot(\bar{c}+\bar{g})\} + a\bar{d} + a\bar{c} + a\bar{g}$ ... (DeM)

$= \bar{a} + [\overline{(b\bar{c})}\cdot\overline{(e+\bar{d})}] \cdot \{[\bar{d}\cdot(\bar{a}+\bar{b})]\cdot(\bar{c}+\bar{g})\} + a\bar{d} + a\bar{c} + a\bar{g}$ ... (DeM)

$= \bar{a} + [(\bar{b}+c)\cdot(\bar{e}\cdot d)] \cdot [(\bar{a}\cdot\bar{d} + \bar{b}\cdot\bar{d})\cdot(\bar{c}+\bar{g})] + a\bar{d} + a\bar{c} + a\bar{g}$ .... (DeM); (D1)

$= \bar{a} + a\bar{d} + a\bar{c} + a\bar{g} + [(\bar{b}+c)\cdot(\bar{e}\cdot d)\cdot(\bar{a}\cdot\bar{d} + \bar{b}\cdot\bar{d})\cdot(\bar{c}+\bar{g})]$

7. Derive the equations for y and z from the following table and simplify the answer as much as possible. Then implement y and z using a minimum number of 2-input and 3-input gates (note: inverters (bubbles) don't count). Also identify the function obtained by y and z (eg. is the function an 'and', 'xor', 'or', 'majority gate' etc.?)

3 marks

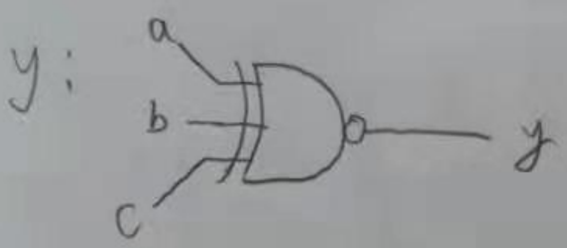
c	b	a	y	z
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	0

$$y = \overline{a \oplus b \oplus c}$$

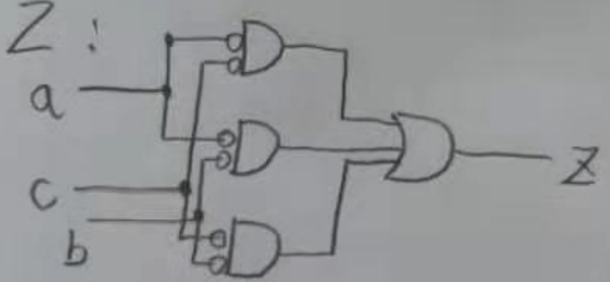
$$z = \bar{a}\bar{c} + \bar{a}\bar{b} + \bar{c}\bar{b}$$

$$= (\overline{a+c}) + (\overline{a+b}) + (\overline{c+b})$$

$$= \overline{(a+c)(a+b)(c+b)}$$



1 gate



or

4 gates