This test paper has two parts and total of 40 marks.

Part I has 6 multiple choice questions. Part II has 2 long answer questions.

It cannot be taken from the examination room.

Only nonprogrammable calculators are allowed. Duration: 50 Minutes.

NAME : **STUDENT NO :**

PART I: Multiple Choice Questions. No partial marks. Circle the correct answer.

[3] **1)** Let $A = \begin{bmatrix} 1 & a & b \\ 0 & -1 & 0 \\ 0 & 0 & b \end{bmatrix}$. For what values of a and b, $A^2 = I_3$?

a) $a \in R$ and $b = -1$

b) $a \in R$ and $b = 1$

c) $a = 0$ and $b = 0$

d) $a = 0$ and $b = 1$

[3] **2)** Let $T : R^2 \to R^2$ be the linear transformation which rotates vectors $\pi/6$ radians counterclockwise. Which of the following matrices is the standard matrix of T?

a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **b)** $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **c)** $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **d)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

[3] **3)** Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}$. Find $(AB^{-1})^{-1}$.

a) $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ **b)** $\frac{1}{2}\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$ **c)** $\frac{1}{2}\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ **d)** $\frac{1}{10}\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

[3] **4)** If $(A^{-1}+4I)^T = \begin{bmatrix} 2 & 8 \\ 1 & 3 \end{bmatrix}$, what is A?

a) $\frac{1}{6}\begin{bmatrix} 1 & -1 \\ -8 & 2 \end{bmatrix}$ **b)** $\frac{1}{6}\begin{bmatrix} 1 & 1 \\ 8 & 2 \end{bmatrix}$ **c)** $\frac{-1}{6}\begin{bmatrix} 1 & 1 \\ 8 & 2 \end{bmatrix}$ **d)** $\frac{1}{2}\begin{bmatrix} -1 & 8 \\ 1 & -2 \end{bmatrix}$

[3] **5)** Let T be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$.

What is $T\left(2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right)$?

a) $\begin{bmatrix} 16 \\ -18 \end{bmatrix}$ **b)** $\begin{bmatrix} -18 \\ 16 \end{bmatrix}$ **c)** $\begin{bmatrix} -16 \\ 18 \end{bmatrix}$ **d)** $\begin{bmatrix} 18 \\ -16 \end{bmatrix}$

[3] **6)** Which of the following linear transformations is invertible:

a) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 8x+2y \\ 4x+y \end{bmatrix}$

b) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x+2y \\ 2x-4y \end{bmatrix}$

c) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-4y \\ 4x-16y \end{bmatrix}$

d) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+2y \\ -2x+4y \end{bmatrix}$

PART II: Long answer questions. Show all your work.

[11] **1)** Let T be the linear transformation given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ x+y+2z \\ 2x+y+2z \\ y+z \end{bmatrix}$.

a) Find the standard matrix of T.

b) Find a vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ whose image under T is the vector $\begin{bmatrix} 2 \\ 5 \\ 3 \\ 3 \end{bmatrix}$.

c) Is T one-to-one? Explain your answer.

d) Is T onto? Explain your answer.

[11] **2)** Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 4 \\ 1 & 4 & 0 \end{bmatrix}$.

a) Find the inverse of the matrix A.

b) Write down the elementary matrix corresponding to each row operation that you performed in part a).

c) Write both A and A^{-1} as a product of elementary matrices.