# Quiz 2 – Fall 2019.

1. Is the vector $[5.2 \;\; 7.6]$ a convex combination of $u = [10 \;\; 6]$ and $v = [4 \;\; 8]$? Prove your answer.

Total: 5

Does their exist a value $\lambda \in (0, 1)$ such that $\boxed{z = \lambda u + (1-\lambda) v}$ ? 1

$$\begin{bmatrix} 5.2 \\ 7.6 \end{bmatrix} = \lambda \begin{bmatrix} 10 \\ 6 \end{bmatrix} + (1-\lambda) \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

First equation:

$$\boxed{5.2 = 10\lambda + 4(1-\lambda).} \quad 1$$

$$= 6\lambda + 4.$$

$$\Rightarrow 6\lambda = 1.2$$

$$\boxed{\lambda = 0.2} \quad 1/2$$

Second equation:

$$\boxed{7.6 = 6\lambda + 8(1-\lambda)} \quad 1$$

$$= -2\lambda + 8$$

$$2\lambda = 0.4$$

$$\boxed{\lambda = 0.2} \quad 1/2$$

1/2

Since both the equations are satisfied for the same value of $\lambda$, it follows that $\begin{bmatrix} 5.2 \\ 7.6 \end{bmatrix}$ is a convex combination of $u$ and $v$. 1/2.

**Question**: Consider the function $f(x_1, x_2) = -x_1 x_2$ .

a.) Using theorem I of concave (convex) functions, determine whether the function is concave, convex, strictly concave, strictly convex, or neither.

b.) What are the implications of your answer for a stationary point of the function?

**Answer:**

**a.)** It is a question here of a function of two variables, $f(x_1, x_2) = -x_1 x_2$ .

Total: 7

Consider two points $\bar{x} = (\bar{x}_1, \bar{x}_2)$ and $\hat{x} = (\hat{x}_1, \hat{x}_2)$ . 1

Evaluate the tangent plane at $\bar{x}$. Then, according to Theorem I of concave/ convex function, one has a relation

$$f(\hat{x}) - f(\bar{x}) \;\; ? \;\; f_1(\bar{x})(\hat{x}_1 - \bar{x}_1) + f_2(\bar{x})(\hat{x}_2 - \bar{x}_2) \, ,$$

2 for the formula in some form.

where ? represents the unknown relation. In the present case, the derivatives are

½ $f_1 = -x_2$ et $f_2 = -x_1$ . ½

Therefore, after replacement, one has

$$-\hat{x}_1\hat{x}_2 + \bar{x}_1\bar{x}_2 \; ? \; -\bar{x}_2(\hat{x}_1 - \bar{x}_1) - \bar{x}_1(\hat{x}_2 - \bar{x}_2)$$

1 for: replacement/substitute for replacement or substitu.

Simplification gives ~~i.e.~~ algebra.

$$(\bar{x}_1 - \hat{x}_1)(\hat{x}_2 - \bar{x}_2) \; ? \; 0$$

1.

When $\bar{x}_1 > \hat{x}_1$ , the expression in parentheses cancels out without changing the relation, yielding

$$\hat{x}_2 \;\; ? \;\; \bar{x}_2 .$$

1.

Thus, there are two possibilities in this case: $\hat{x}_2 > \bar{x}_2$ and the function is convex; $\hat{x}_2 < \bar{x}_2$ and the function is concave. That is, the function is concave in certain regions ~~de~~ of its domain and ~~concave~~ convex in others.

**Alternative answer:** substitute for algebra.

1

If the sign ? is $\geq$ then the function is convex.

If the sign ? is $>$ then the function is strictly convex.

If the sign ? is $\leq$ then the function is concave.

If the sign ? is $<$ then the function is strictly concave.

Alternatively:

If $\bar{x}_1 > \hat{x}_1$ and $\hat{x}_2 \geq \bar{x}_2 \Rightarrow \geq$

... $\hat{x}_2 \leq \bar{x}_2 \Rightarrow \leq$

... $\bar{x}_1 < \hat{x}_1$ and $\hat{x}_2 \geq \bar{x}_2 \Rightarrow \leq$

... $\hat{x}_2 \leq \bar{x}_2 \Rightarrow \geq$

And $>$ $\Rightarrow$ convex, etc.

The function is concave on certain regions of its domain and convex in others.

Total: 2

**b.)** If the function is convex, then the stationary point is a global minimum

If the function is strictly convex, then the stationary point is a unique global minimum

If the function is concave, then the stationary point is a global maximum

If the function is strictly concave, then the stationary point is a unique global ~~minimum~~ max

They should distinguish "global".
Also good if they distinguish "weak" vs "strict." (or "strong").