# MAAE 2400
# Thermodynamics and Heat Transfer

## Review
## (Please note: (1) this review is not comprehensive, and (2) if used out of context may result in errors)

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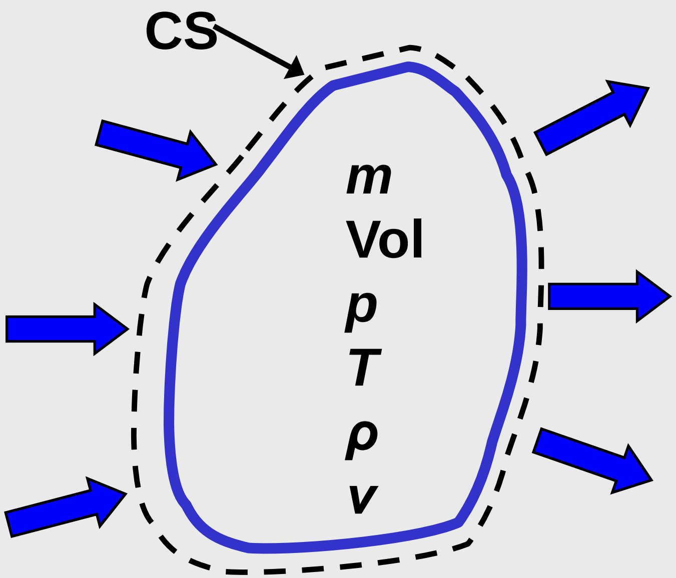
# 1. Conservation of Mass




$$\frac{dm_{CV}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

Continuity equation

$$\frac{dm_{CV}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

For steady state $\frac{dm_{CV}}{dt} = 0$

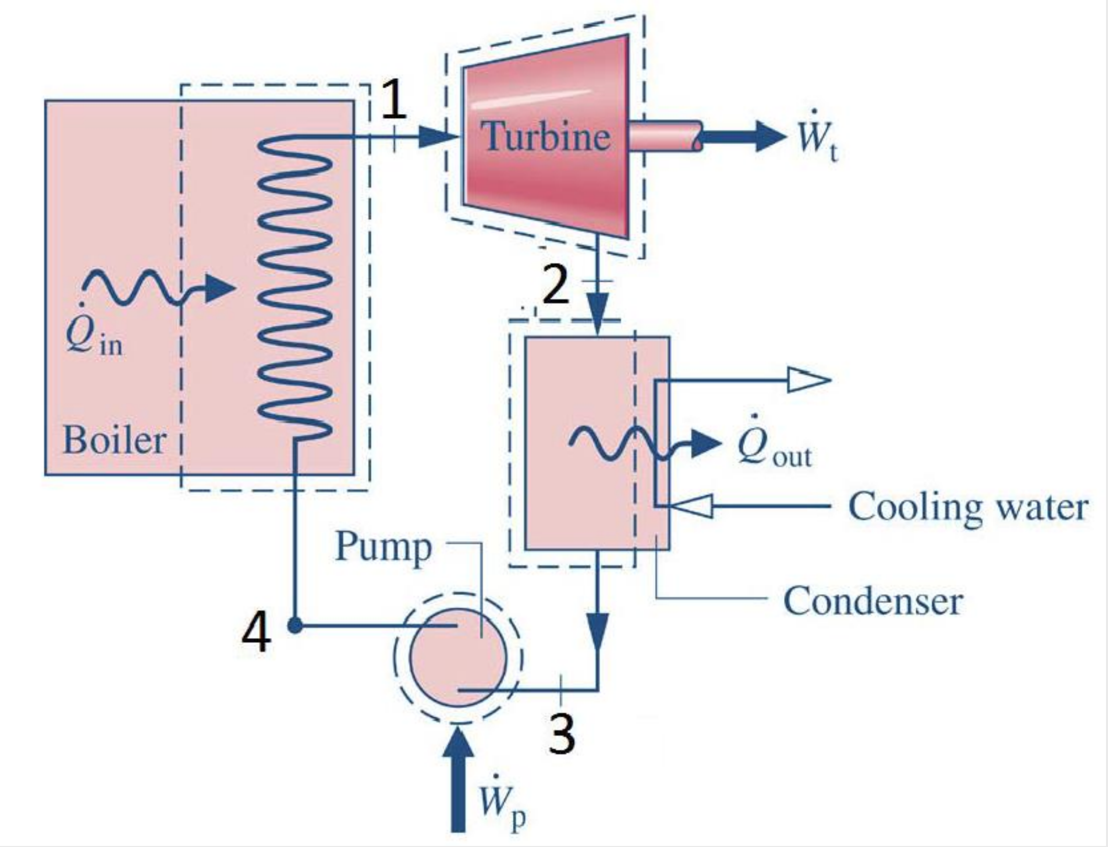
$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

For one-stream in and out of a control volume

$$\dot{m}_{In} = \dot{m}_{Out}$$

$$\dot{m}_{In} = \dot{m}_{Out}$$

Applied to each control volume




$$\dot{m}_1 = \dot{m}_2$$

$$\dot{m}_2 = \dot{m}_3$$

$$\dot{m}_3 = \dot{m}_4$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$$

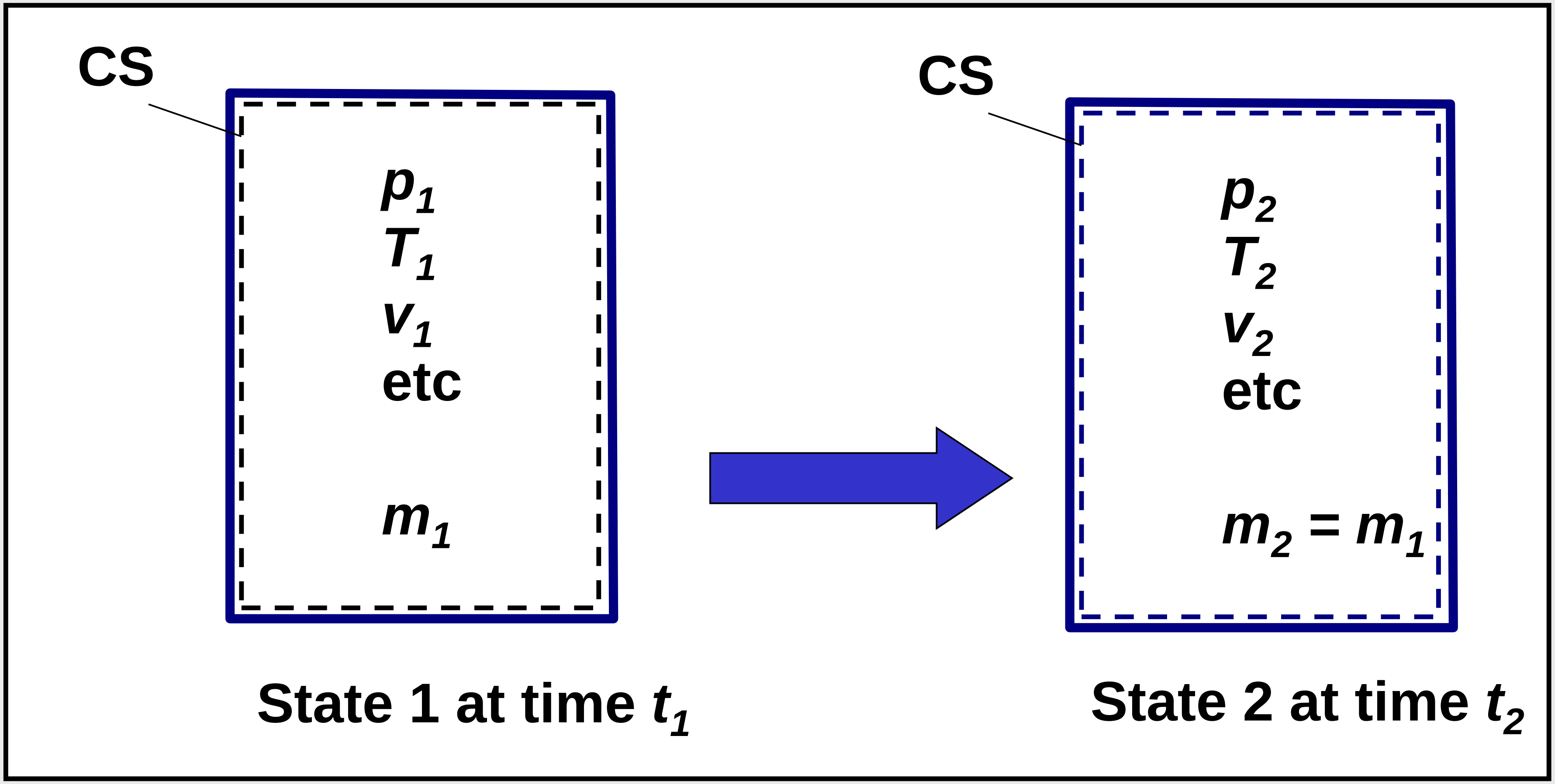
Note: $\dot{m} = \rho(VA) = \frac{(VA)}{v}$

**Could result in**

$$v_1 \neq v_2 \neq v_3 \neq v_4$$

# 2. Conservation of Energy

## 2.1 First Law for Closed Systems



# 2. Conservation of Energy
## 2.1 First Law for Closed Systems

$$E_2 - E_1 = Q_{1-2} - W_{1-2}$$

**Sign Convention:**

**Q > 0 : heat transfer <u>to</u> the system**
**Q < 0 : heat transfer <u>from</u> the system**

**W > 0 : work done <u>by</u> the system**
**W < 0 : work done <u>on</u> the system**

# Other Formulations for the 1st Law of Thermodynamics

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2$$

$$\Delta U + \Delta KE + \Delta PE = {}_1Q_2 - {}_1W_2$$

# Differential Formulations

$$dE = \delta Q - \delta W$$

where

$dE$ : the differential of the total energy of the system

$\delta Q$ : the transfer of energy by heat

$\delta W$ : the transfer of energy by work

# The Time Rate Formulation

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

- **Note: In this case it is more convenient to define Q and W as energy "in" and "out" respectively** (instead of following sign conventions previously introduced)

# Specific Energy Formulation

$$e_2 - e_1 = {}_1q_2 - {}_1w_2$$

where

$e$ : specific energy; $E/m$

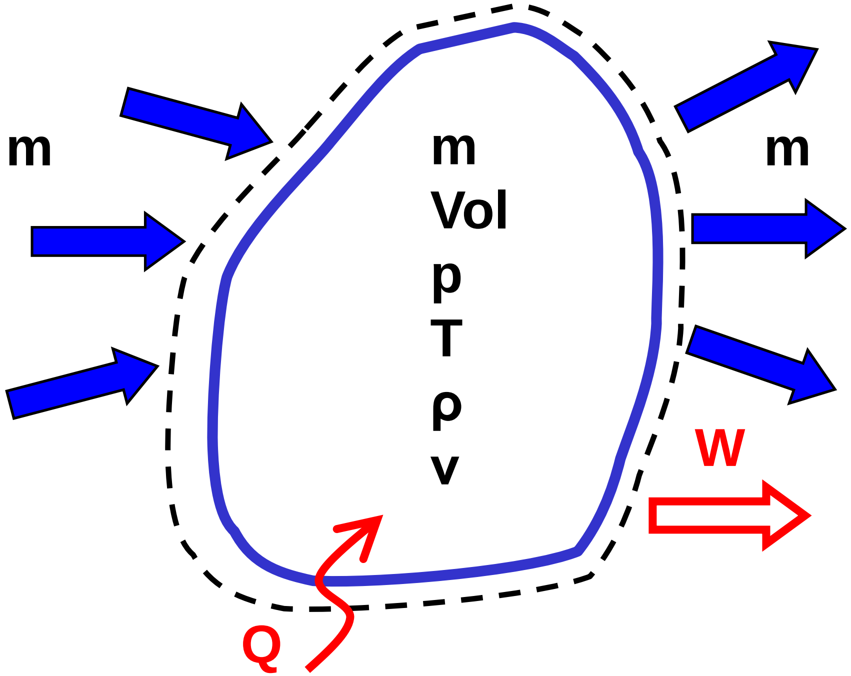
$q$ : specific heat transfer; $Q/m$

$w$ : specific work; $W/m$

($m$ : mass of the system)

# 2. Conservation of Energy

## 2.2 First Law for Open Systems




**During the interval Δt, the change in the total energy inside the CV is given by:**

$$\Delta E_{CV} = Q - W + \sum_i E_i - \sum_e E_e$$

Recall:

$$E = U + KE + PE + etc$$

# The variation of *E* in the CV can be expressed on a rate basis:

$$\frac{dE_{CV}}{dt} = \dot{Q} - \dot{W} + \sum_i \dot{m}_i \cdot e_i - \sum_e \dot{m}_e \cdot e_e$$

$\frac{dE_{CV}}{dt}$ : time rate of change of the energy *contained* within the CV at time t

$\dot{Q}$ : net rate of energy transfer by heat across the CV

$\dot{W}$ : net rate of energy transfer by work across the CV

$\sum_i \dot{m}_i \cdot e_i$ : sum of all energy flow rates entering the CV accompanying mass flow

$\sum_e \dot{m}_e \cdot e_e$ : sum of all energy flow rates exiting the CV accompanying mass flow

# Replacing the energy term by "h", "KE" and "PE"

**The energy balance equation for open system becomes:**

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_i \dot{m}_i \left(h + \frac{V^2}{2} + gz\right)_i - \sum_e \dot{m}_e \left(h + \frac{V^2}{2} + gz\right)_e$$

Where, by convention

$\dot{Q}_{CV} > 0 \quad \Rightarrow$ Heat transfer TO the CV

$\dot{W}_{CV} > 0 \quad \Rightarrow$ Power supplied BY the CV

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_i \dot{m}_i \left(h + \frac{V^2}{2} + gz\right)_i - \sum_e \dot{m}_e \left(h + \frac{V^2}{2} + gz\right)_e$$

Steady state

One inlet and one outlet

$\Delta KE = 0$ ; $\Delta PE = 0$

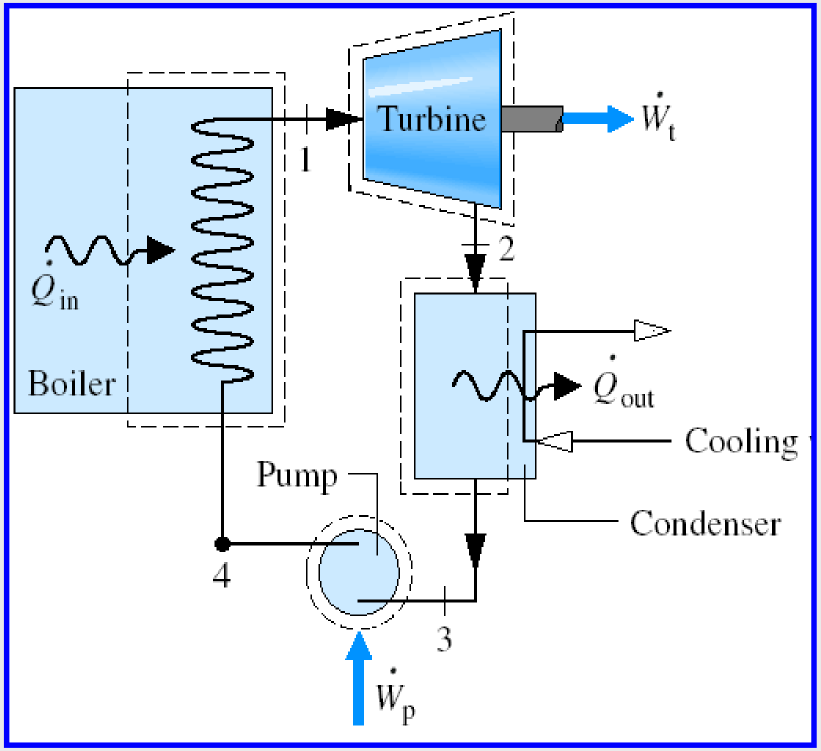
$$0 = \dot{Q} - \dot{W} + \dot{m}(h_{In} - h_{Out})$$

*divide both sides by* $\dot{m}$

$$0 = q - w + (h_{In} - h_{Out})$$

$$0 = q - w + (h_{In} - h_{Out})$$

## RANKINE CYCLE




$$\text{Turbine}: \frac{\dot{W}}{\dot{m}} = w_t = h_1 - h_2$$

$$\text{Condenser}: \frac{\dot{Q}_{out}}{\dot{m}} = q_{out} = h_2 - h_3$$

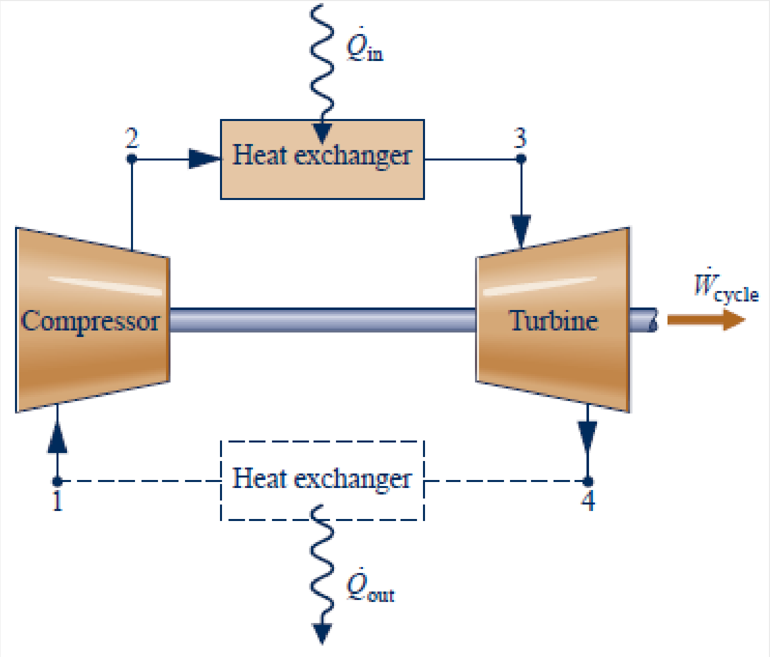
$$\text{Pump}: \frac{\dot{W}_p}{\dot{m}} = w_p = h_4 - h_3$$

$$\text{Boiler}: \frac{\dot{Q}_{in}}{\dot{m}} = q_{in} = h_1 - h_4$$

**Note: Expressions for $q_{out}$ and $w_p$ have been arranged to have positive values in the direction of arrows**

$$0 = q - w + (h_{In} - h_{Out})$$

## BRAYTON CYCLE




$$\text{For } 1-2 \quad |{}_1w_2| = h_2 - h_1$$

$$\text{For } 2-3 \quad |{}_2q_3| = h_3 - h_2$$

$$\text{For } 3-4 \quad |{}_3w_4| = h_3 - h_4$$

$$\text{For } 4-1 \quad |{}_4q_1| = h_4 - h_1$$

1-2 Compression: isentropic

2-3 Heat added at constant pressure

3-4 Expansion: isentropic

4-1 Heat rejected at constant pressure

***NOTE: w's and q's are written such that all have <u>positive</u> values***

$$0 = q - w + (h_{In} - h_{Out})$$

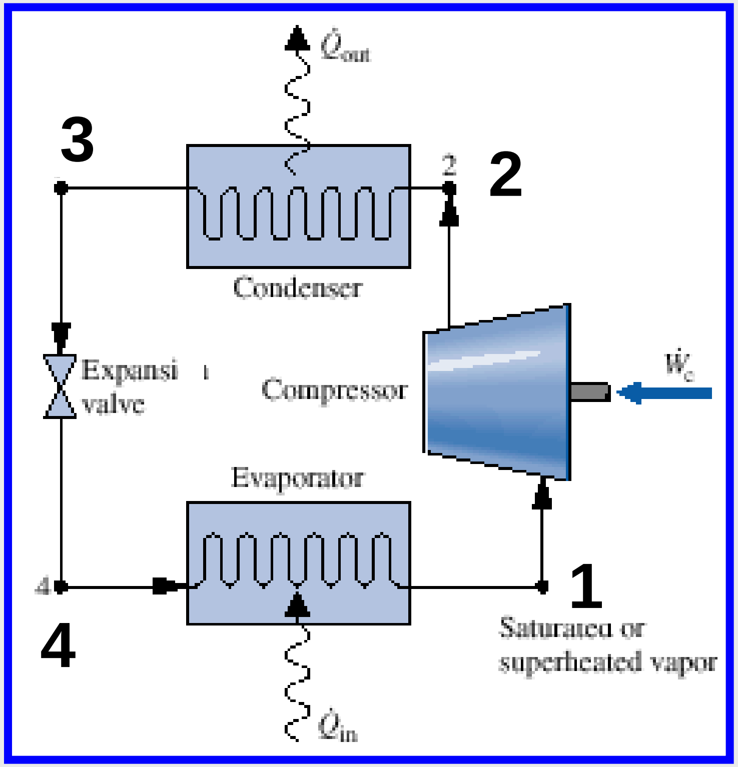
## Vapor Compression Refrigeration CYCLE

$$For\ 1-2:\ |w_{1-2}| = h_2 - h_1$$

$$For\ 2-3:\ |q_{2-3}| = h_2 - h_3$$

$$For\ 3-4:\quad h_4 = h_3$$

$$For\ 4-1:\ |q_{4-1}| = h_1 - h_4$$




*NOTE: w's and q's expressions have been rearranged to be all positive*

# 3. Thermodynamics Cycles

## Power Cycles

For a cycle, the initial and final states are the same

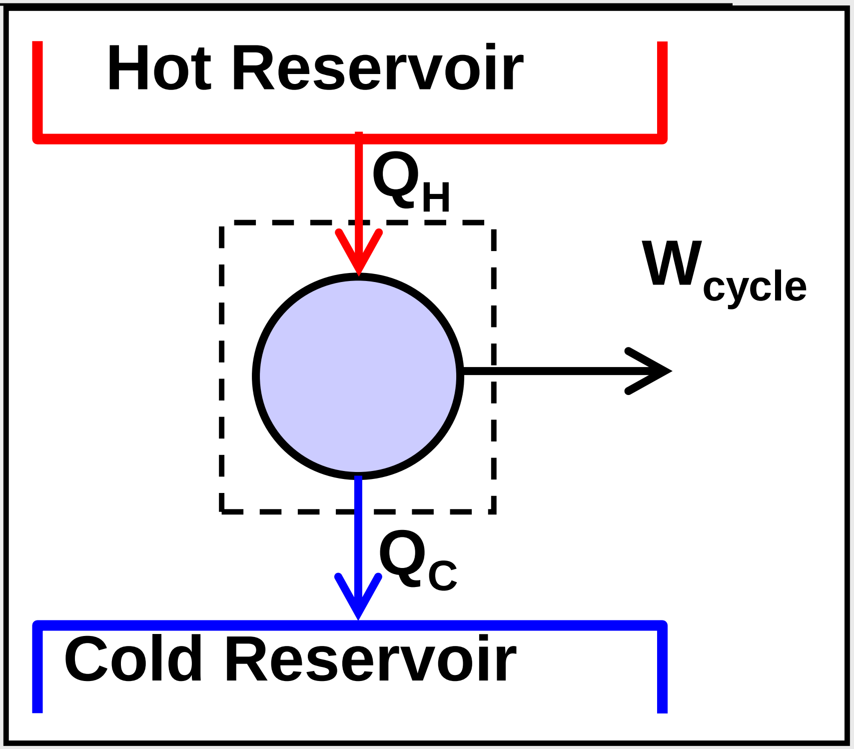
In a cycle, the same mass (called the working fluid) is continuously recirculated

$$\Delta E_{cycle} = Q_{Net} - W_{Net}$$

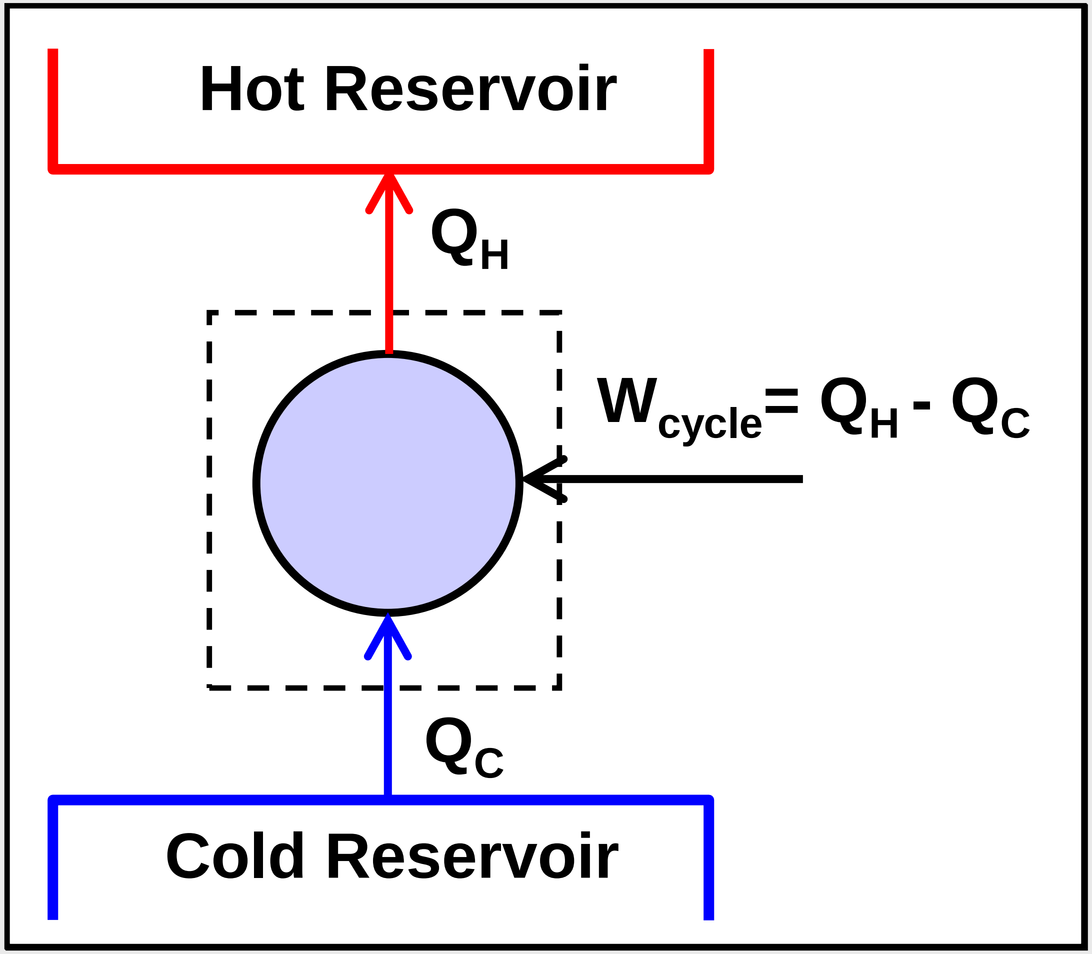
$$0 = Q_{Net} - W_{Net}$$

$$W_{Net} = Q_{Net}$$

$$W_{cycle} = Q_H - Q_C$$



## Refrigeration and Heat Pump Cycles



## In the rate form:

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$$

$$(\dot{W}_{turbine} - \dot{W}_{pump})$$

**For vapor power cycles**

$$(\dot{W}_{turbine} - \dot{W}_{compressor})$$

**For gas power cycles**

## In the specific energy form:

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$$

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{Q}_H}{\dot{m}} - \frac{\dot{Q}_C}{\dot{m}}$$

$$w = q_H - q_C$$

# 4. Entropy Equation
## 4.1 Closed Systems

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

$$\left\{\text{Entropy change}\right\} = \left\{\text{Entropy transfer by heat transfer}\right\} + \left\{\text{Entropy generation}\right\}$$

**Note:**

$\sigma$ = Entropy produced during the process
$\sigma = 0$ No irreversibilities present during the process
$\sigma > 0$ Irreversibilities present during the process
$\sigma < 0$ Impossible

# Other Form of Entropy Equation for Closed Systems

$$S_2 - S_1 \cong \sum_j \frac{Q_j}{T_j} + \sigma$$

where

$Q_j$ : Heat transfer across part j of the system boundary

$T_j$ : Temperature of part j of the system boundary

# 4. Entropy Equation
## 4.2 Open Systems

**Entropy equation is expressed in similar manner**

$$\underbrace{\frac{dS_{CV}}{dt}}_{\substack{\text{Rate of}\\ \text{Entropy Change}\\ \text{in CV}}} = \underbrace{\underbrace{\sum_j \frac{\dot{Q}_j}{T_j}}_{\substack{\text{Heat Transfer}\\ \text{Contribution}}} + \underbrace{\sum_i \dot{m}_i \cdot s_i - \sum_e \dot{m}_e \cdot s_e}_{\substack{\text{Convection}\\ \text{Contribution}}}}_{\text{Rate of Entropy Transfer}} + \underbrace{\dot{\sigma}_{CV}}_{\substack{\text{Rate}\\ \text{of}\\ \text{Entropy}\\ \text{Production}}}$$

# 4. Entropy Equation
# 4.2 Open Systems (continued)

$$\underbrace{\frac{dS_{CV}}{dt}}_{\substack{\text{Rate of} \\ \text{Entropy Change} \\ \text{in CV}}} = \underbrace{\underbrace{\sum_j \frac{\dot{Q}_j}{T_j}}_{\substack{\text{Heat Transfer} \\ \text{Contribution}}} + \underbrace{\sum_i \dot{m}_i \cdot s_i - \sum_e \dot{m}_e \cdot s_e}_{\substack{\text{Convection} \\ \text{Contribution}}}}_{\text{Rate of Entropy Transfer}} + \underbrace{\dot{\sigma}_{CV}}_{\substack{\text{Rate} \\ \text{of} \\ \text{Entropy} \\ \text{Production}}}$$

$\dot{\sigma}_{CV}$ = *Rate of entropy production*

$\dot{\sigma}_{CV} > 0$ *Irreversibilities present*

$\dot{\sigma}_{CV} = 0$ *No irreversibilities present*

$\dot{\sigma}_{CV} < 0$ *Impossible*

# Cycle Thermal Efficiency

**Cycle Performance** | **Carnot (Ideal) Cycle Performance**

$$\eta_{th} = \frac{W_{cycle}}{Q_H} = 1 - \frac{Q_C}{Q_H} \;\Rightarrow\; \eta_{th,rev} = 1 - \frac{T_C}{T_H}$$ **Power Cycle**

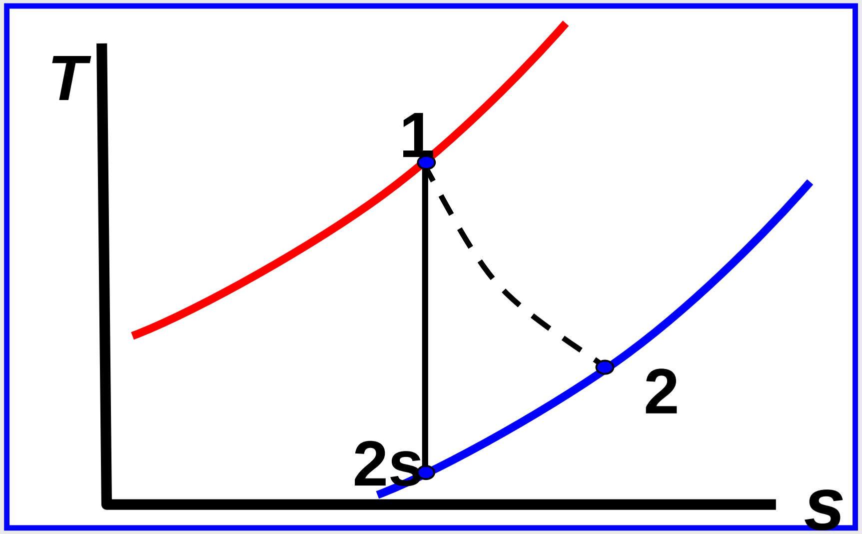
$$\beta_{th} = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_c} \;\Rightarrow\; \beta_{th,rev} = \frac{T_C}{T_H - T_C}$$ **Refrigeration Cycle**

$$\gamma_{th} = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_c} \;\Rightarrow\; \gamma_{th,rev} = \frac{T_H}{T_H - T_C}$$ **Heat Pump Cycle**

# Isentropic efficiency
# & Effectiveness

# Turbine Isentropic Efficiency

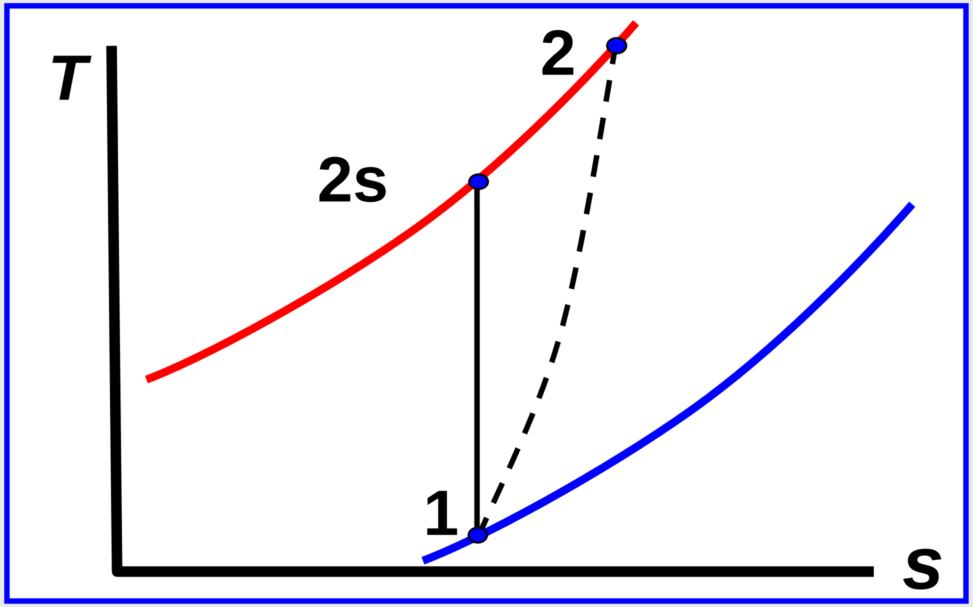
$$\eta_{turb} \equiv \frac{Actual\ Output}{Ideal\ Output}$$




$$\eta_{turb} = \frac{w_{1-2}}{w_{1-2s}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

# Compressor Isentropic Efficiency

$$\eta_{Comp} \equiv \frac{Ideal\ Work\ Input}{Actual\ Work\ Input}$$



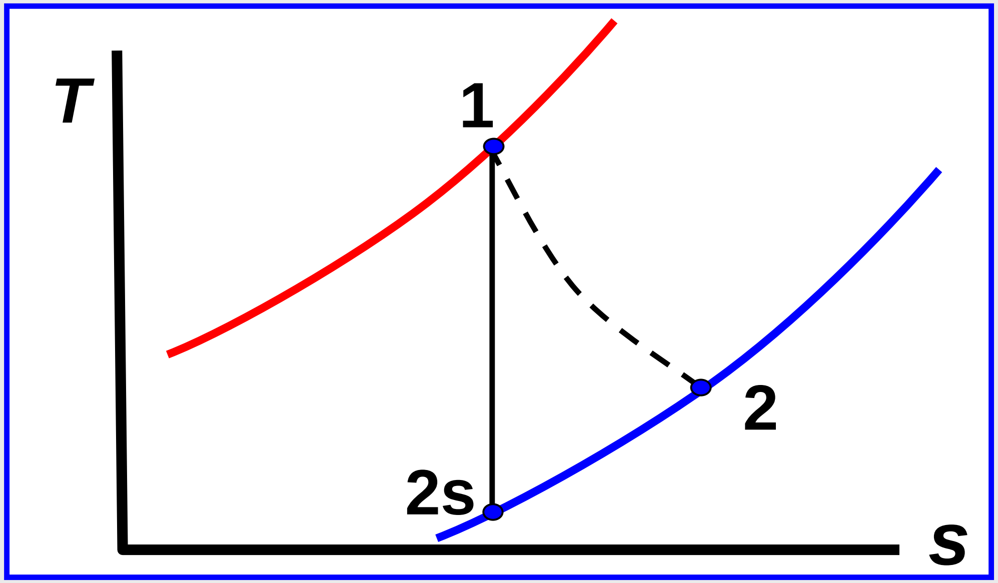

$$\eta_{turb} = \frac{w_{1-2_s}}{w_{1-2}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

# Nozzle Isentropic Efficiency

$$\eta_{Nozzle} \equiv \frac{Actual\ \Delta\text{KE}}{\text{Ideal}\ \Delta\text{KE}}$$

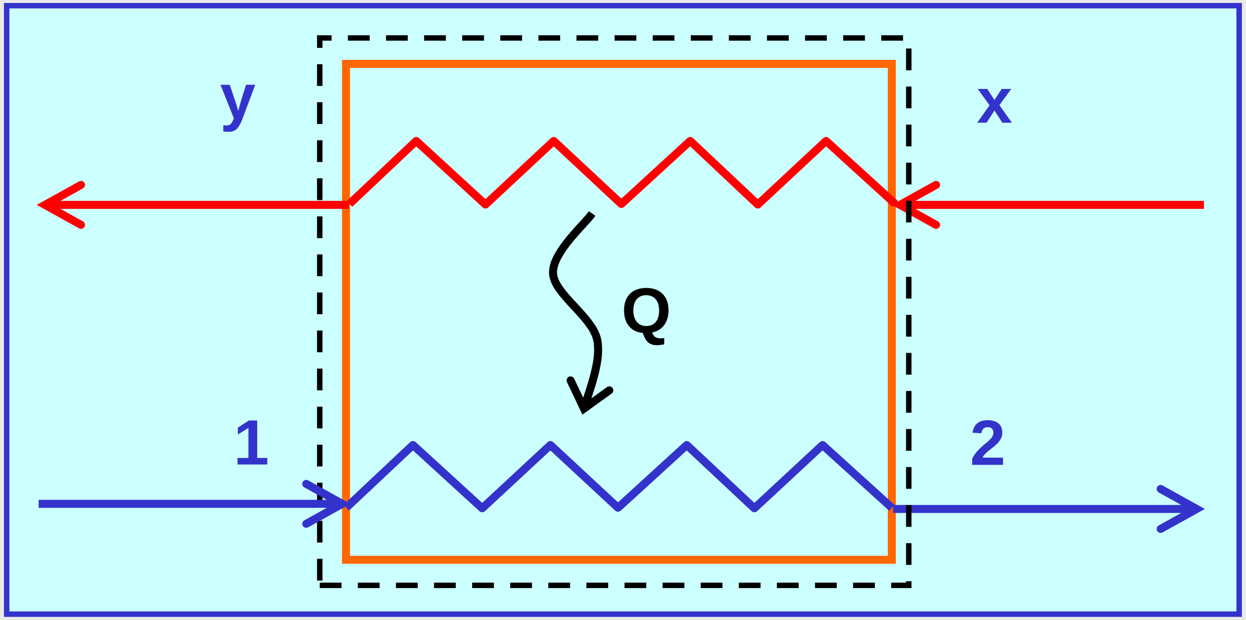
$$\eta_{Nozzle} = \frac{V_2^2 - V_1^2}{V_{2s}^2 - V_1^2}$$

$$\eta_{Nozzle} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

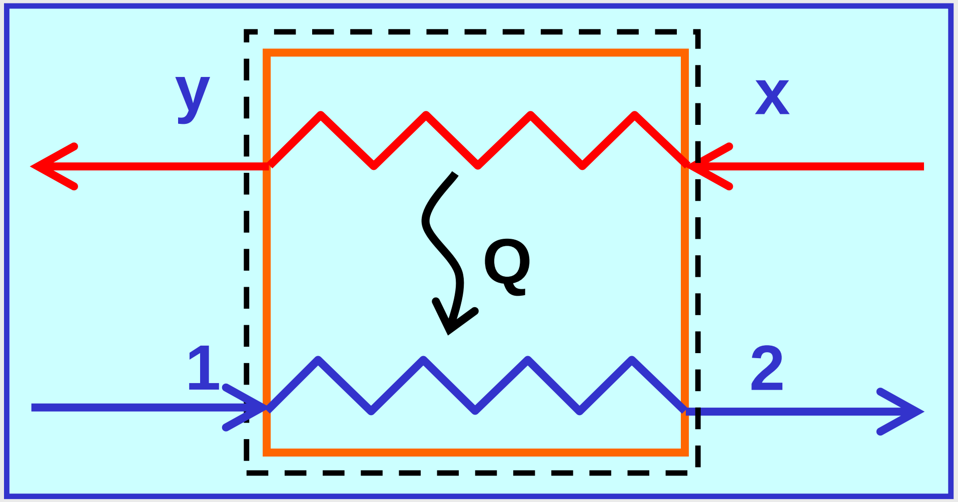


# Heat Exchanger (HX) Effectiveness

$$\eta_{HX} = \frac{Actual\ Temp.\ Change}{Maximum\ Theoretical\ Temp.\ Change}$$



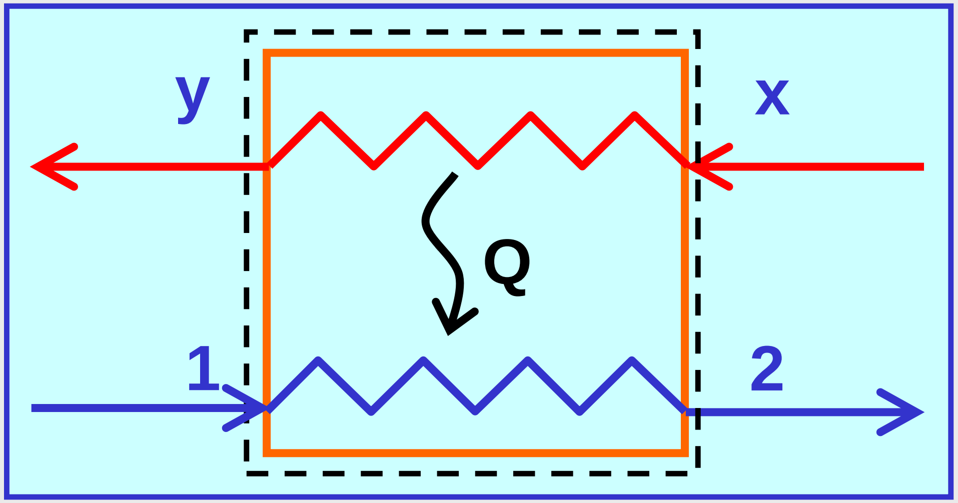
# If HX is viewed as to heat stream 1-2




$$\eta_{HX} = \frac{Actual\ T_{rise}}{Theoretical\ Maximum\ T_{rise}}$$

$$\eta_{HX} = \frac{T_2 - T_1}{T_x - T_1}$$

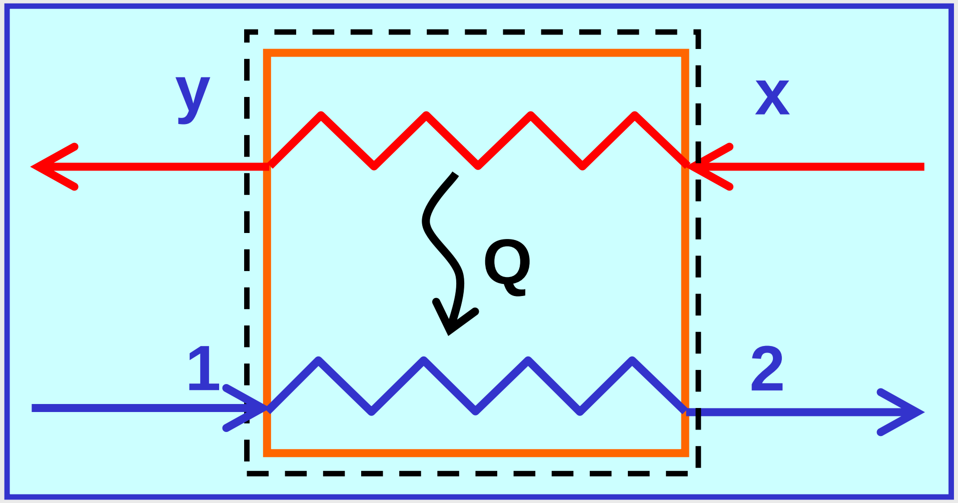
# If HX is viewed as to cool stream x-y




$$\eta_{HX} = \frac{Actual\ T_{drop}}{Theoretical\ Maximum\ T_{drop}}$$

$$\eta_{HX} = \frac{T_x - T_y}{T_x - T_1}$$

# Energy balance in HX




$$\dot{m}_{Cold\ stream}(h_2 - h_1) = \dot{m}_{Hot\ stream}(h_x - h_y)$$

**(If there is no heat losses to the surrounding)**

# Regenerator Effectiveness

**(Regenerative gas turbines)**

**Regenerator**

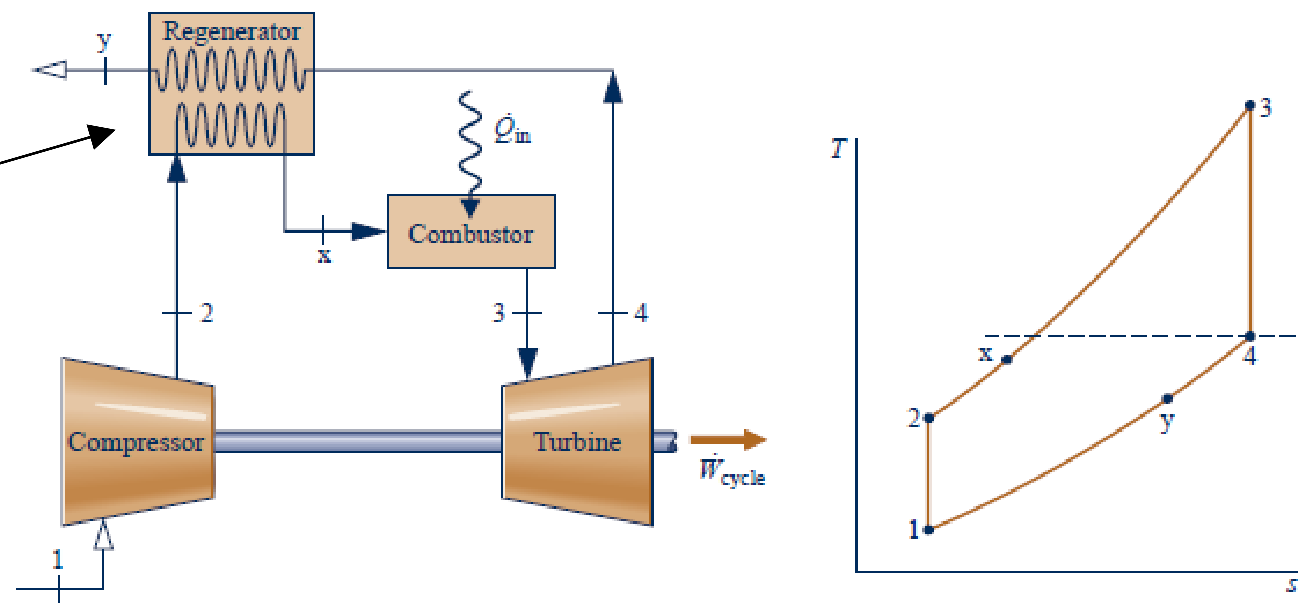



Fig. 9.14 Regenerative air-standard gas turbine cycle.

$$\eta_{reg.} = \frac{h_x - h_2}{h_4 - h_2}$$

$$\eta_{reg.} = \frac{h_4 - h_y}{h_4 - h_2}$$

# Regenerator Effectiveness

**(Regenerative gas turbines)**

Regenerator

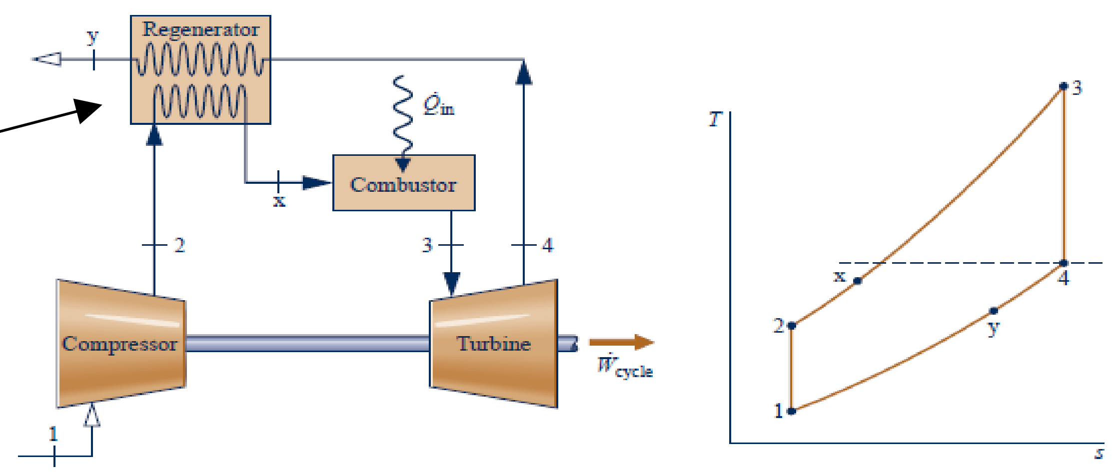



**Fig. 9.14** Regenerative air-standard gas turbine cycle.

$$(h_x - h_2) = (h_4 - h_y)$$

**(Assuming no heat losses)**

# Ideal Work Relationships for Open System

$$\frac{\dot{W}}{\dot{m}} = -v\left(p_2 - p_1\right)$$

Open system, Steady - state, 1 Stream, Reversible, Incompressible

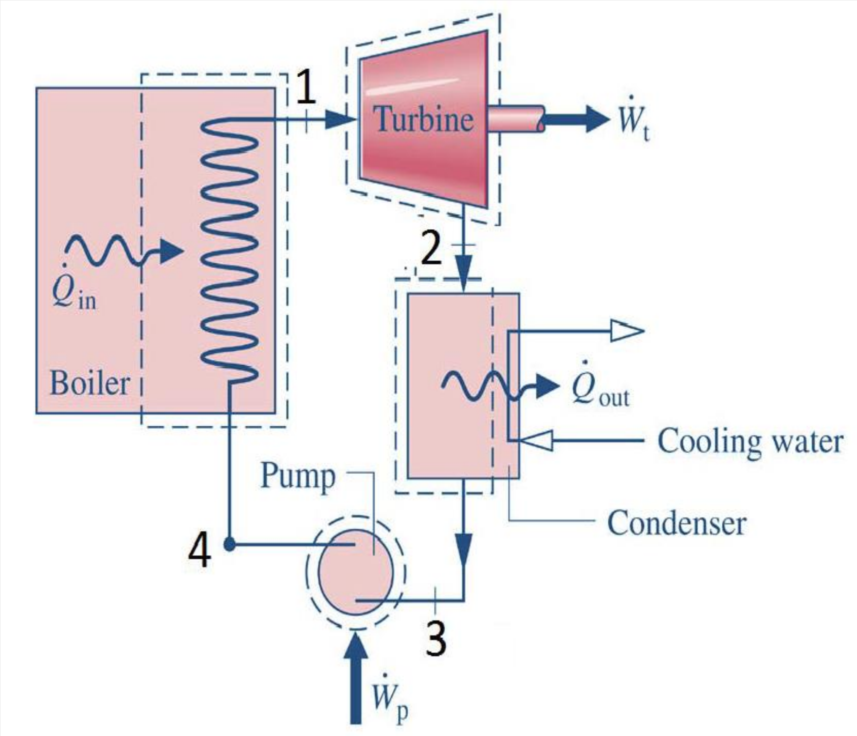
**Applied to "ideal" pumps for example**

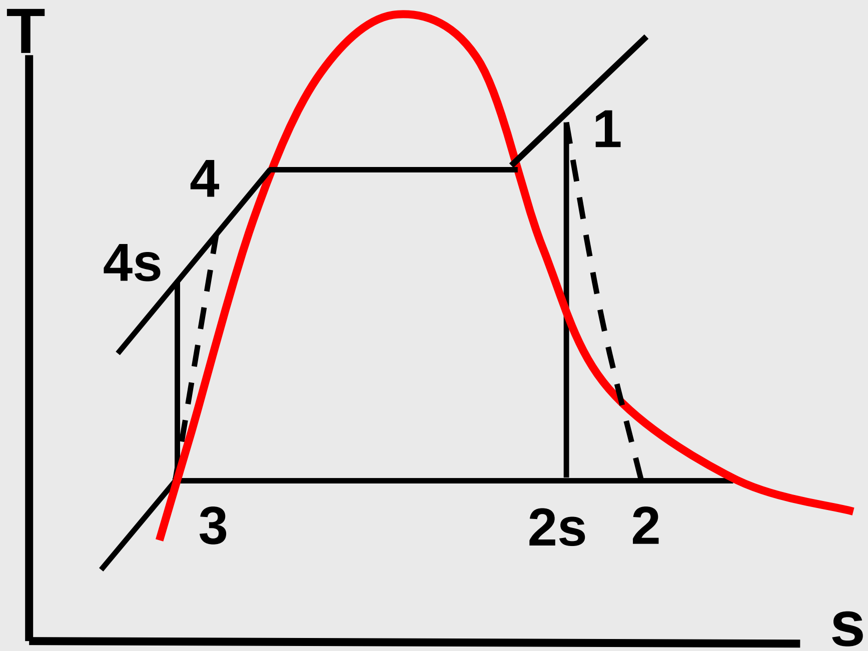
$$\frac{\dot{W}_{p_s}}{\dot{m}} = w_{p_s} = v(p_2 - p_1)$$

(Magnitude of pump work)

# Analyses of Thermodynamics Cycles

# Rankine Cycle



# *One way to evaluate "$h_{4s}$"*

***For isentropic pump process*** $3 \rightarrow 4_s$

$$\left\{\begin{aligned} w_{ps} &= v_3(p_4 - P_3) \\ w_{ps} &= h_{4s} - h_3 \end{aligned}\right.$$

$$h_{4s} = h_3 + v_3(p_4 - P_3)$$

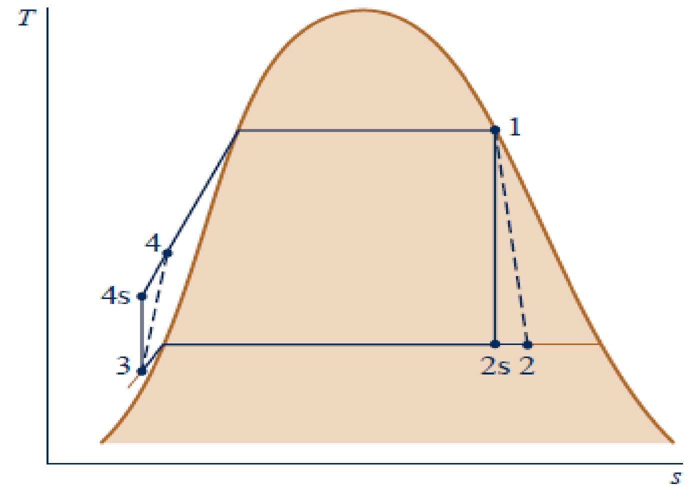
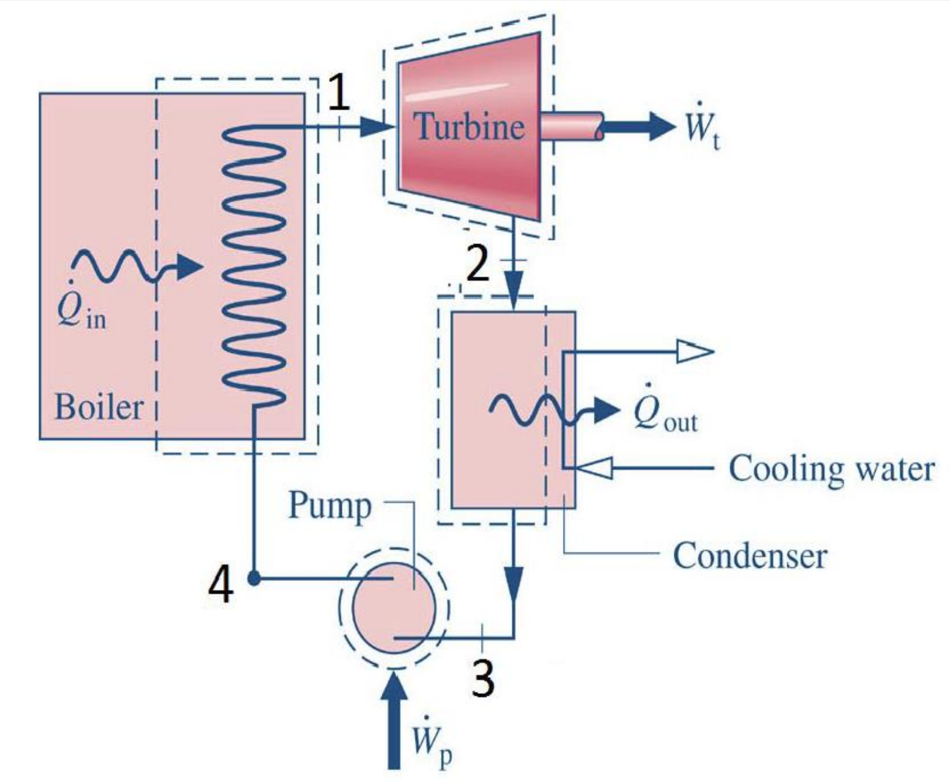



Fig. 8.6 Temperature–entropy diagram showing the effects of turbine and pump irreversibilities.

# Energy balance for Rankine cycles



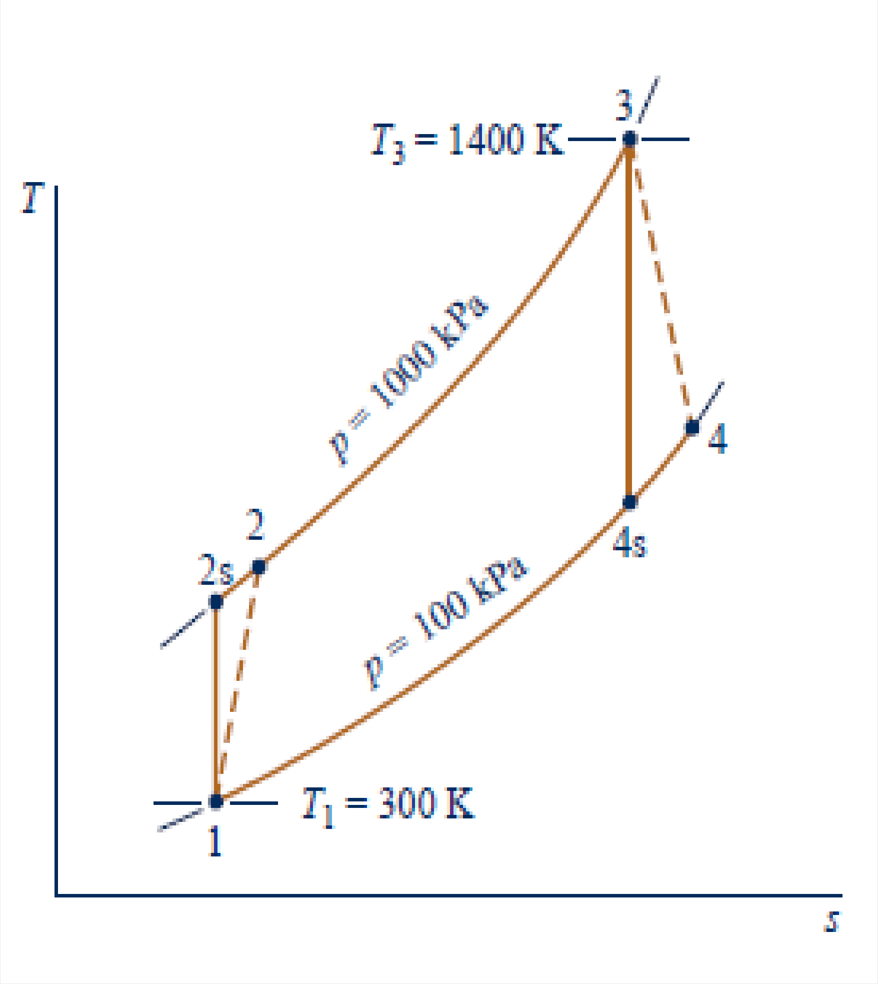

$$\dot{W}_{cycle} = \dot{Q}_{In} - \dot{Q}_{Out}$$

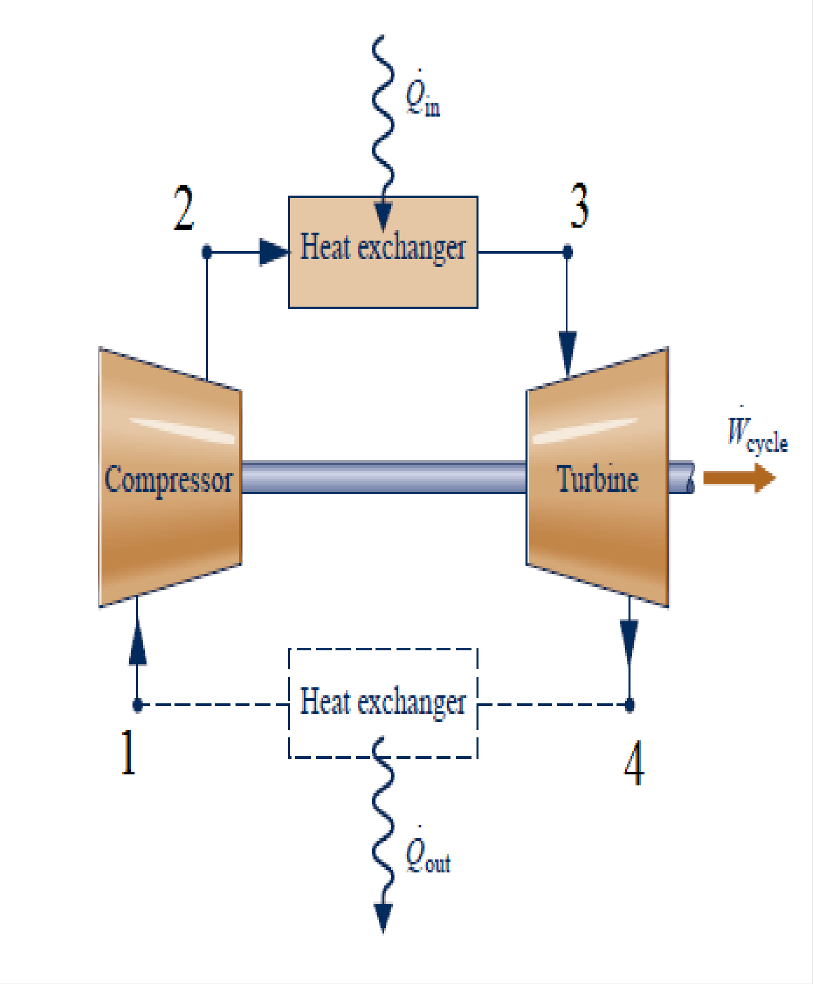
$$\dot{W}_t - \dot{W}_p = \dot{Q}_{In} - \dot{Q}_{Out}$$

$$\frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = \frac{\dot{Q}_{In}}{\dot{m}} - \frac{\dot{Q}_{Out}}{\dot{m}}$$

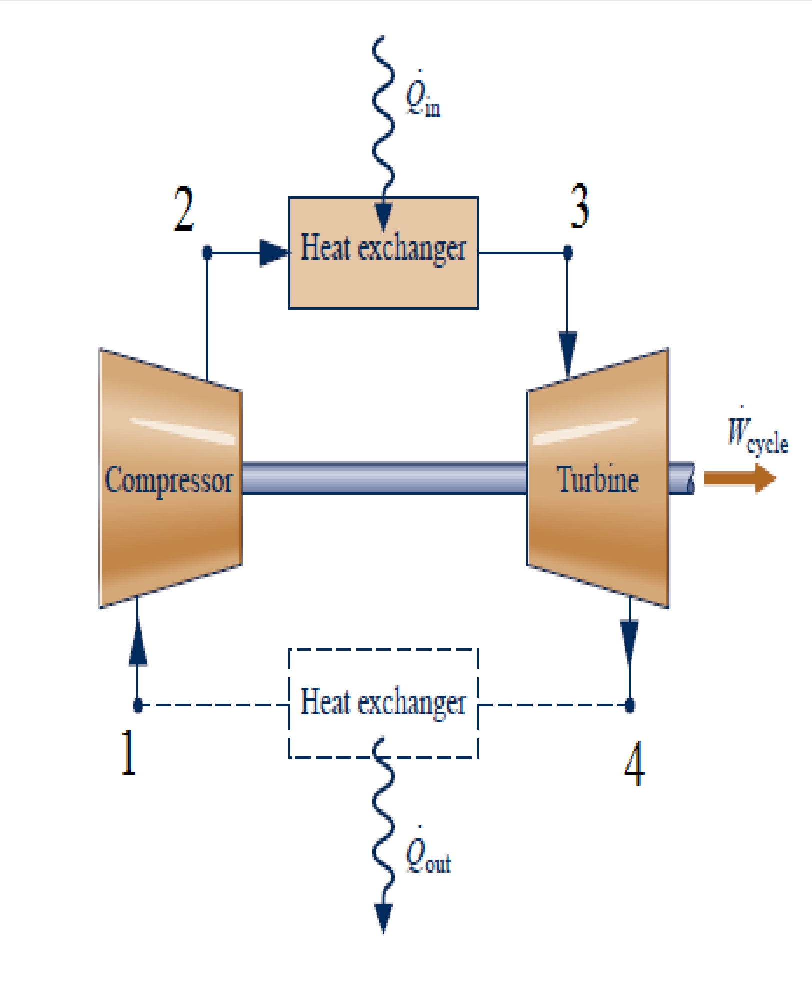
$$w_t - w_p = q_{In} - q_{Out}$$

# Brayton Cycle



# Energy balance for Brayton cycles




$$\dot{W}_{cycle} = \dot{Q}_{In} - \dot{Q}_{Out}$$

$$\dot{W}_t - \dot{W}_c = \dot{Q}_{In} - \dot{Q}_{Out}$$

$$\frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_c}{\dot{m}} = \frac{\dot{Q}_{In}}{\dot{m}} - \frac{\dot{Q}_{Out}}{\dot{m}}$$

$$w_t - w_c = q_{In} - q_{Out}$$

# Vapor-Compression Refrigeration Systems