Winter, 2018 **Math*1030** **Final Exam**

Last name _____ First name _____
(PRINT)

Student # _____ Signature _____

Instructor: M. R. Garvie April 14, 2018

INSTRUCTIONS

1. This is a closed book examination. Scientific and/or graphing calculators are allowed. The test is 2 hours long. You may use blank areas of this exam booklet for rough work. Make sure you have a complete exam booklet (as errors sometimes occur during printing).

2. The test consists of 40 equally-weighted (independent) multiple choice questions. Answer the multiple choice questions on the computer score sheet (**circle your answers on the exam paper**). In each question choose the answer that best fits the question.

3. Fill in the computer score sheet in pencil; make sure you include your name and student ID number, and your **email address** (letters before the @ symbol), but you don't need a section number. Also fill in your name and student number at the top of this exam booklet.

4. The abbreviation 'd.p.' stands for 'decimal places', e.g. $\pi = \mathbf{3.14}$ (2 d.p.) means that π is given to 2 decimal places.

5. Trig functions should always be evaluated in **radians**.

6. **Hand in the entire exam booklet and your computer score sheet.**

1. The solution of $x^2 \leq 4$ is

(A) $-2 \leq x \leq 2$
(B) $x \leq 2$
(C) $x \leq -2$ or $x \geq 2$
(D) $x \leq \pm 2$
(E) $-2 < x < 2$

2. The function $y = 3 - \dfrac{1}{x+2}$ has

(A) a horizontal asymptote $y = 3$, but no vertical asymptote
(B) a vertical asymptote $x = -2$, but no horizontal asymptote
(C) a horizontal asymptote $y = -3$, but no vertical asymptote
(D) a vertical asymptote $x = -2$ and a horizontal asymptote $y = 3$
(E) no asymptotes

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3. The domain of $y = 5x^2 - 10$ is

(A) $\mathbb{R}$
(B) $(2, +\infty)$
(C) The empty set $\emptyset$
(D) $(-\infty, 2)$
(E) $(-\infty, 2) \cup (2, +\infty)$

4. Let $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x}$, then the domain of $(f \circ g)(x)$, i.e. $D_{f \circ g}$ is

(A) $(0, +\infty]$
(B) $(-\infty, 0)$
(C) $[0, +\infty)$
(D) $(0, +\infty)$
(E) the empty set $\emptyset$

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5. The function $f(x) = x^{2/3}$ is

(A) odd
(B) neither odd nor even
(C) even
(D) undefined at $x = 0$
(E) odd and even

6. The function $y = |x|$ is

(A) neither one-to-one nor many-to-one
(B) one-to-one
(C) one-to-many
(D) many-to-one
(E) many-to-none

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7. If $f(x) = \dfrac{1}{x}$ then $f^{-1}(x)$ is

(A) x
(B) $\ln(x)$
(C) $\dfrac{1}{y}$
(D) xy
(E) $\dfrac{1}{x}$

8. The period of $\cos\left(\dfrac{1}{2}x\right)$ is

(A) π
(B) 4π
(C) 2π
(D) $1/2$
(E) $\pi/2$

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9. The function $y = 2 + \dfrac{1}{x+1}$ corresponds to

(A) a shift of $y = 1/x$ in the x-direction by $+1$ units followed by a shift in the y-direction by $+2$ units

(B) a shift of $y = 1/x$ in the x-direction by -1 units followed by a shift in the y-direction by $+2$ units

(C) none of the above

(D) a shift of $y = 1/x$ in the x-direction by -1 units followed by a shift in the y-direction by -2 units

(E) a shift of $y = 1/x$ in the x-direction by $+1$ units followed by a shift in the y-direction by -2 units

10. The function $y = \sqrt{x-1}$ is

(A) strictly monotonic increasing on $[-1, \infty)$

(B) neither strictly monotonic increasing nor strictly monotonic decreasing

(C) strictly monotonic increasing on $[1, \infty)$

(D) strictly monotonic decreasing on $[1, \infty)$

(E) strictly monotonic decreasing on $(-\infty, 1]$

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11. The maximum value of $f(x) = |(x-1)(x-2)|$ on $[1, 2]$ is

(A) 0
(B) $1/4$
(C) $-1/4$
(D) $3/4$
(E) $1/2$

12. The **range** of $f(x) = 2^x - 1$ is given by

(A) $R_f = \mathbb{R}$
(B) $R_f = (-\infty, -1)$
(C) $R_f = [-1, +\infty)$
(D) $R_f = (-1, +\infty)$
(E) $R_f = (-\infty, -1]$

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13. The function $y = x|x|$ simplifies to

(A) x^2

(B) $\begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

(C) $\begin{cases} 2x & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$

(D) $\begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

(E) $\begin{cases} -x^2 & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$

14. The equation of the straight line passing through the points $(1, 2)$ and $(3, 4)$ is

(A) $y = -x + 3$
(B) $y = x - 3$
(C) $y = x - 2$
(D) $y = x$
(E) $y = x + 1$

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15. A firm selling tinned salmon finds that the total cost in dollars of producing and selling x tins is given by

$$C(x) = 3x + 3000.$$

The firm charges $\$6$ per tin. What is the break-even quantity?

(A) $\$1000$
(B) The firm will never break-even
(C) 1500 tins
(D) $\$1500$
(E) 1000 tins

16. The quadratic function $y = -(x-2)^2 - 1$ has

(A) a minimum value of 2 at $x = -1$
(B) a maximum value of -1 at $x = 2$
(C) a maximum value of 1 at $x = 2$
(D) a minimum value of -1 at $x = 2$
(E) a maximum value of -2 at $x = -2$

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17. The solution of $9^{-3x} \cdot 9^{x} = 27$ is

(A) $x = -3/4$
(B) $x = 2$
(C) $x = -1/2$
(D) $x = 3/4$
(E) $x = -1$

18. The solution of $7 + 15e^{1-3x} = 10$ is (to 2 d.p.)

(A) -0.87
(B) 0.87
(C) -0.20
(D) 0.27
(E) $+0.20$

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19. In the Exponential Growth Model (base e) if the growth rate is $\mathbf{4.5\%}$ (units day^{-1}) how long does it take for the initial quantity to double? (Answer given to $\mathbf{2}$ d.p.)

(A) $\mathbf{0.15}$ days
(B) $\mathbf{0.03}$ days
(C) $\mathbf{6.69}$ days
(D) $\mathbf{15.40}$ days
(E) $\mathbf{20.12}$ days

20. As x gets closer and closer to $\mathbf{0}$ the function $\dfrac{\ln(x+1)}{x}$ gets closer and closer to

(A) an undefined quantity
(B) $\mathbf{0.99998}$
(C) $+\infty$
(D) $\mathbf{1}$
(E) $\mathbf{0}$

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21. Consider the recursively defined sequence

$$a_1 = 1, \quad a_2 = 2, \quad a_3 = 3, \quad a_{n+1} = \frac{a_n}{1 + a_{n-1}} + a_{n-2}, \quad n = 3, 4, 5, \ldots$$

The 4th term is

(A) $7/2$
(B) 1
(C) 2
(D) 1.5
(E) 3

22. The 30th term of the Arithmetic sequence

$$\left\{\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \ldots\right\}$$

is given by

(A) $1/39366$
(B) $61/6$
(C) $65/6$
(D) $21/2$
(E) 0

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23. $\lim\limits_{n\to\infty}\left(\dfrac{6n^2+2n+1}{-n^2+n+2}\right)$ is

(A) $-\infty$
(B) 0
(C) $-1/6$
(D) $1/2$
(E) -6

24. $\lim\limits_{n\to\infty}\left(\dfrac{3}{2}\right)^n =$

(A) $-\infty$
(B) $+\infty$
(C) 0
(D) 1
(E) $2/3$

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25. $\sum_{i=1}^{\infty} 2(0.3)^i =$

(A) $+\infty$
(B) $\mathbf{20/7}$
(C) $\mathbf{6/7}$
(D) limit does not exist
(E) $\mathbf{5}$

26. The future value of an investment of $\mathbf{\$1000}$ earning $\mathbf{3\%}$ simple interest p.a. for $\mathbf{200}$ days is

(A) $\mathbf{\$7000}$
(B) $\mathbf{\$2643.84}$
(C) $\mathbf{\$1643.84}$
(D) $\mathbf{\$1016.44}$
(E) $\mathbf{\$16.44}$

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27. The present value of $\$2098$ due in 4 months at 4.3% p.a. compounded three times per year is

(A) $\$1834.99$
(B) $\$2398.71$
(C) $\$2011.51$
(D) $\$2068.35$
(E) $\$2128.07$

28. The effective interest rate corresponding to 10% compounded continuously is (to 2 d.p.)

(A) 0.11
(B) 11.23%
(C) 10.32%
(D) 1.01%
(E) 10.52%

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29. If $f(x) = \dfrac{(x-3)(x+1)}{(x-3)}$ then $\lim\limits_{x \to 3} f(x)$ is

(A) 4
(B) 0
(C) 3
(D) $+\infty$
(E) undefined

30. If

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x+1 & \text{if } x > 1 \end{cases},$$

then $\lim\limits_{x \to 1} f(x)$

(A) $= +\infty$
(B) $= 2$
(C) $= 0$
(D) does not exist
(E) $= 1$

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31. Consider the piecewise defined function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 5 & \text{if } x > 2 \end{cases}$$

Then $\lim_{x \to 2^-} f(x)$ ('left hand limit') is

(A) undefined (B) $+\infty$ (C) $= 3$

(D) $= 5$ (E) $= 4$

32. The function $f(x) = \dfrac{2^x}{x-4}$ is

(A) discontinuous at $x = 0$

(B) discontinuous at $x = 4$

(C) continuous at $x = 4$

(D) discontinuous for all x

(E) continuous for all x

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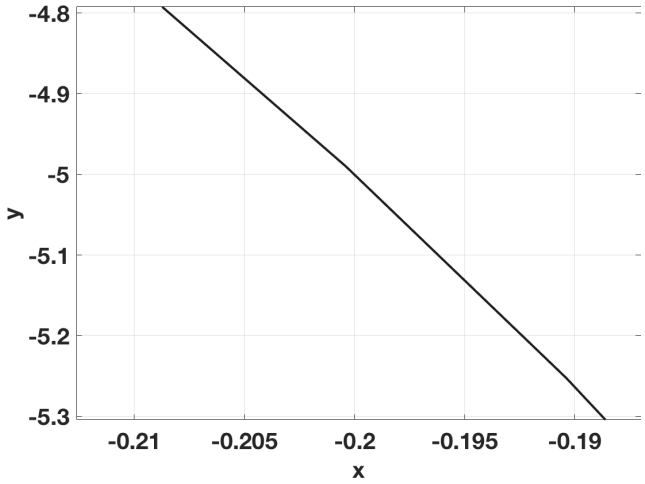
33. The function $f(x) = \sqrt{3 - x}$ is

(A) discontinuous everywhere
(B) continuous everywhere
(C) continuous from the left at $x = 3$
(D) continuous from the right at $x = 3$
(E) undefined at $x = 3$

34. After 'zooming in' to the graph of a function we obtain the approximate straight line segment illustrated below.



The approximate instantaneous rate of change of the function at $x = -0.2$ is

(A) 25 (B) -0.1 (C) -0.04
(D) -25 (E) 0.04

35. The difference quotient for the function $f(x) = 2x + 1$ simplifies to

(A) $2 + h$
(B) 1
(C) h
(D) 2
(E) $2h/h$

36. Given $y = \dfrac{x+1}{x-1}$ then $\dfrac{dy}{dx} =$

(A) $\dfrac{2}{(x-1)^2}$ (B) $-\dfrac{2}{(x-1)^2}$ (C) $-\dfrac{2}{(x+1)^2}$

(D) 0 (E) $\dfrac{2}{(x+1)^2}$

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37. If $y = \ln(x^3 + 1)$ then $\dfrac{dy}{dx} =$

(A) $3x^2 \ln(x^3 + 1)$ (B) $\ln(3x^2)$ (C) $\dfrac{3x^2}{x^3 + 1}$

(D) $3x^2 e^{x^3+1}$ (E) $\dfrac{1}{x^3 + 1}$

38. Suppose the revenue (in dollars \$) earned from selling q items of a product is given by

$$R(q) = \frac{20,000q - 2q^2}{4,000}.$$

Then the marginal revenue from selling $q = 1000$ items is

(A) $\$4,500$
(B) $-\$6$
(C) $\$4$
(D) $\$0.00$
(E) $\approx \$3.9995$

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39. Consider the function

$$f(x) = x^4 - 2x^2 + 3.$$

Application of the First Derivative Test yields that we have

(A) no critical point at $x = 1$
(B) a point of inflection at $x = 1$
(C) a local maximum at $x = 1$
(D) no local extrema
(E) a local minimum at $x = 1$

40. Consider the function

$$f(x) = x^3 - 9x^2 + 24x.$$

Then application of the Second Derivative Test yields that

(A) $f(2)$ is a local maximum and the test gives no information at $x = 4$
(B) $f(2)$ is a local minimum and $f(4)$ is a local maximum
(C) $f(2)$ is a local maximum and $f(4)$ is a local minimum
(D) $f(4)$ is a local minimum and the test gives no information at $x = 2$
(E) The test yields no information for the given function

END OF TEST