Discrete Mathematics for Computing

Elizabeth Maltais

MAT1348A Assignment 1

due: Tuesday, January 28 before 9:00 pm

Instructions

- This is a **group assignment**. Form a team of **at most 4 members**, and submit **one paper for the team**. Each team member will receive the same mark for the assignment. (Teams may consist of 1, 2, 3, or 4 students). All team members must be registered to the same section of MAT1348.
- You must hand in your assignment **in the assignment submission box, just outside STEM complex room 207** on or before **Tuesday, January 28 by 9:00 pm**. **Late assignments will not be accepted.** Make sure you submit your assignment in the box labelled with your professor's name and the course code and section in which you are registered.
- Please print this document, including the cover page, **staple it**, and write your answers in the space provided. You may write on the backs of pages or insert extra pages if necessary, so long as the assignment is clearly organized, with solutions in the same order as the questions, and securely stapled.
- You must hand in a legible, organized and properly stapled assignment. If it is too difficult to read your solutions, then you (and your entire team) may get zero.
- Solutions must include all relevant steps and justifications where appropriate. If you only write the final answer without explanation, then you may not receive full marks.
- The maximum points possible = 20 points

  †Your signature indicates that you made substantial contributions to solving **all** of the problems of this assignment in a collaborative effort **with your entire team**, and that, individually, **you understand what has been submitted** well enough that you could reproduce each of the solutions herein on short notice. You are also signing to indicate that your other team members worked on these solutions with you and that **you think all members of the team understand the solutions submitted herein.**

Team Members (in alphabetical order, based on family name):

SOLUTIONS

1.	Family Name:	Student Number:
	First Name:	†Signature:
2.	Family Name:	Student Number:
	First Name:	†Signature:
3.	Family Name:	Student Number:
	First Name:	†Signature:
4.	Family Name:	Student Number:
	First Name:	†Signature:

Q1-a Let M, H, and S denote the following propositional variables: [6 points]

M : "The moon is full" H : "The wolf will howl." S : "The wolf stalks its prey."

Translate each of the following sentences into compound propositions using the above propositional variables, logical connectives, and parentheses when appropriate. *Parentheses have been added to some sentences for clarification; otherwise, follow the conventions of precedence of logical connectives.*

P_1: A necessary condition for the wolf to howl is that the moon is full. $H \rightarrow M$

P_2: The wolf does not stalk its prey unless the moon is full $\neg S \vee M$

$(\equiv \neg M \rightarrow \neg S \equiv S \rightarrow M)$

P_3: (The wolf will howl whenever the moon is full) only if the wolf does not stalk its prey.

$(M \rightarrow H) \rightarrow \neg S$

P_4: The moon is not full, but (the wolf will howl if and only if it stalks its prey).

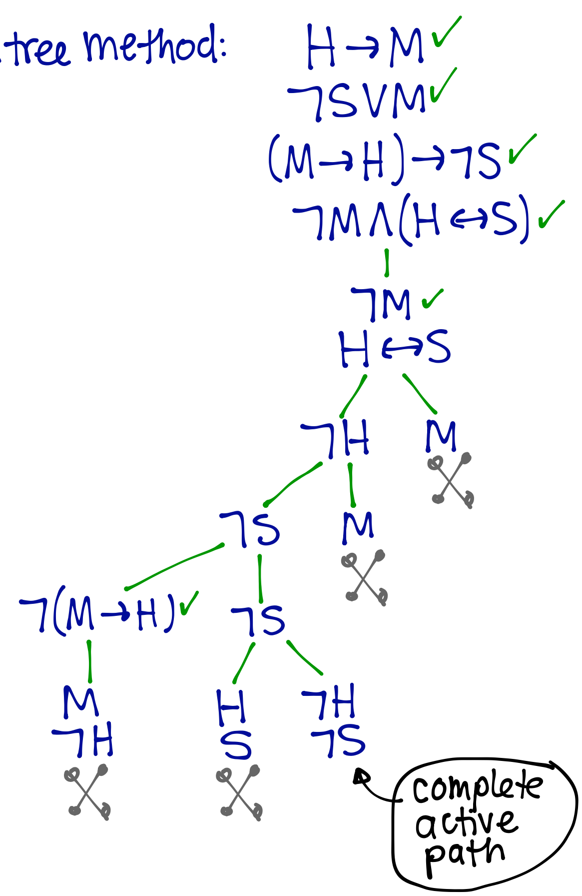
$\neg M \wedge (H \leftrightarrow S)$

Q1-b Using the method of your choice, determine whether or not the set $\{P_1, P_2, P_3, P_4\}$ of propositions from above is **consistent**. Show your work and give **all** truth assignments of the variables M, H, and S that support your answer, with a brief explanation.

Is the set $\{P_1, P_2, P_3, P_4\}$ **consistent**? *Circle:* **YES** **NO**

Justification:

(truthtree method:



All 4 propositions are true when M is F, H is F, and S is F.

H	M	S	P_1: $H \to M$	P_2: $\neg S \vee M$	P_3: $(M \to H) \to \neg S$	P_4: $\neg M \wedge (H \leftrightarrow S)$
T	T	T	T	T	F	F
T	T	F	T	T	T	F
T	F	T	F	F	F	T
T	F	F	F	T	T	F
F	T	T	T	T	T	F
F	T	F	T	T	T	F
F	F	T	T	F	F	F
F	F	F	T	T	T	T

Since there exists at least one truth assignment that makes all 4 propositions true, the set $\{P_1, P_2, P_3, P_4\}$ is consistent.

There is exactly one truth assignment to support this conclusion:

all 4 propositions are true when: H is F, M is F, and S is F.

Q2. For this question, you will prove in two ways that the propositions X and Y, given below, are **logically equivalent**.

$$X: \quad (a \rightarrow b) \rightarrow \neg(c \vee d) \qquad\qquad Y: \quad (a \wedge \neg b) \vee (\neg c \wedge \neg d)$$

(a) [3 points] Use an appropriate **truth table** to verify that $X \equiv Y$. Make sure you briefly explain how your truth table demonstrates that $X \equiv Y$.

a	b	c	d	$(a\rightarrow b)$	$c\vee d$	$\neg(c\vee d)$	X	$a\wedge\neg b$	$\neg c\wedge\neg d$	Y
T	T	T	T	T	T	F	F	F	F	F
T	T	T	F	T	T	F	F	F	F	F
T	T	F	T	T	T	F	F	F	F	F
T	T	F	F	T	F	T	T	F	T	T
T	F	T	T	F	T	F	T	T	F	T
T	F	T	F	F	T	F	T	T	F	T
T	F	F	T	F	T	F	T	T	F	T
T	F	F	F	F	F	T	T	T	T	T
F	T	T	T	T	T	F	F	F	F	F
F	T	T	F	T	T	F	F	F	F	F
F	T	F	T	T	T	F	F	F	F	F
F	T	F	F	T	F	T	T	F	T	T
F	F	T	T	T	T	F	F	F	F	F
F	F	T	F	T	T	F	F	F	F	F
F	F	F	T	T	T	F	F	F	F	F
F	F	F	F	T	F	T	T	F	T	T

Since truth values of X always match the truth values of Y, for every possible truth assignment, we conclude that $X \leftrightarrow Y$ must be a tautology $\therefore X \equiv Y$ (by definition)

(Q2 continued)

(b) [3 points] Use the Laws from the Table of Logical Equivalences to prove that $X \equiv Y$.

You may use **one (and only one!)** law at each step, and you must write the name of the law used at each step.

$(a \to b) \to \neg(c \vee d) \equiv \neg(a \to b) \vee \neg(c \vee d)$ Implication Law

$\equiv \neg(\neg a \vee b) \vee \neg(c \vee d)$ Implication Law

$\equiv (\neg\neg a \wedge \neg b) \vee \neg(c \vee d)$ De Morgan's Law

$\equiv (\neg\neg a \wedge \neg b) \vee (\neg c \wedge \neg d)$ De Morgan's Law

$\equiv (a \wedge \neg b) \vee (\neg c \wedge \neg d)$ Double Negation Law

(c) [1 point] Write a disjunctive normal form (DNF) for X. You do not need to justify your answer for this part.

Notice that Y is in DNF and $X \equiv Y$

$\therefore$ $(a \wedge \neg b) \vee (\neg c \wedge \neg d)$ is a DNF for X.

BONUS[+2 points] Suppose we meet 3 inhabitants, namely A, B, and C, of the Island of Knights & Knaves (each of whom is either a knight or a knave).

A says: "I know where the gold is hidden if and only if at most one of us is a knight."

B says: "A knows where the gold is hidden only if I am a knave."

C says: "A is a knave, but A knows where the gold is hidden."

What is possible to conclude from this? For each question below, circle the most appropriate response. You do not need to justify your answer for this bonus question, but all of your answers must be correct in order to receive the bonus points.

What type of inhabitant is A? *Circle:* Knight Knave It's impossible to determine A's type.

What type of inhabitant is B? *Circle:* Knight Knave It's impossible to determine B's type.

What type of inhabitant is C? *Circle:* Knight Knave It's impossible to determine C's type.

Does A know where the gold is hidden? *Circle:* Yes No It's impossible to determine this.

a	b	c	g	A says $g \leftrightarrow$ (at most one knight)	B says $g \to \neg b$	C says $\neg a \wedge g$	
T	T	T	T	F			
T	T	T	F	T	T	F	
T	T	F	T	F			
T	T	F	F	T	T	F	$\leftarrow$ possible
T	F	T	T	F			
T	F	T	F	T	T		
T	F	F	T	T	T		
T	F	F	F	F			
F	T	T	T	F	F		
F	T	T	F	T			
F	T	F	T	T			
F	T	F	F	F	T	F	$\leftarrow$ possible
F	F	T	T	T			
F	F	T	F	F	T		
F	F	F	T	T			
F	F	F	F	F	T		

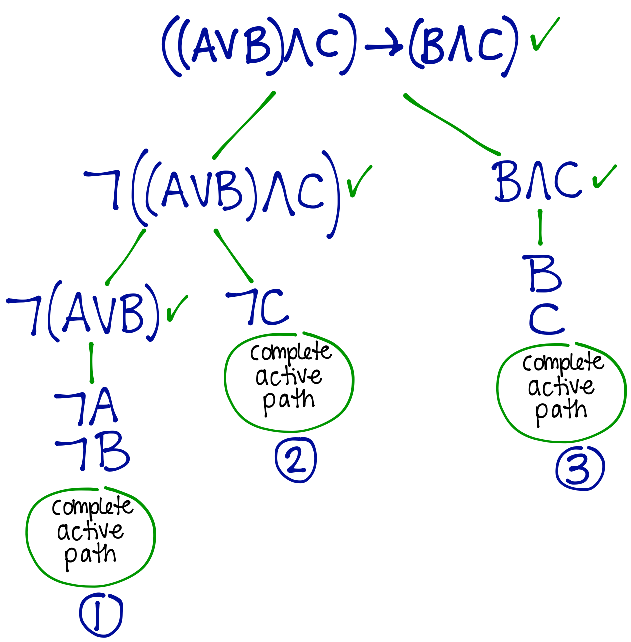
In both possible scenarios B is a Knight, C is a knave, A does not know where gold is hidden. We cannot determine A's type.

Q3. [7 points] Let P be the following compound proposition consisting of atoms A, B, and C:

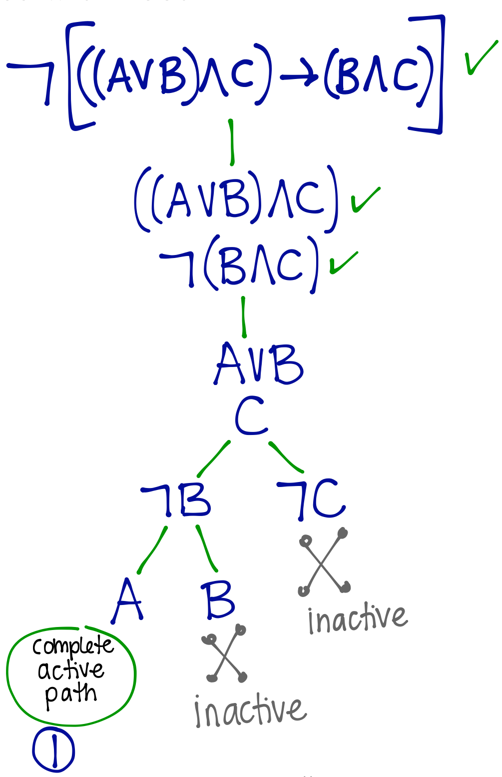
$$P: \quad \big((A \vee B) \wedge C\big) \rightarrow (B \wedge C)$$

For this question, you will grow two truth trees. For each tree, you must use the *official* branching rules and apply the branching rules to the propositions as they are written (i.e. do not use logical equivalences to change the propositions in your tree – stick to the official branching rules, seen in class). Clearly label each path as active(alive) or inactive (dead).

Tree 1. Complete truth tree with root P



Tree 2. Complete truth tree with root $\neg P$



Q3 continued.

3a. Is P a **tautology**? *Circle:* **YES** **NO**

Briefly explain. Your explanation must make reference to Tree 1 or Tree 2 explicitly, its root, and any relevant paths in the tree.

In Tree 2, there is at least one complete active path which tells us that the root of Tree 2, namely $\neg P$, can be true

Since $\neg P$ can be true, P itself can be false $\therefore$ P is not a tautology

In Tree 2, the complete active path tells us

$\neg P$ is T when A is T, B is F, C is true

$\therefore$ P is F when A is T, B is F, C is true

3b. Is P a **contradiction**? *Circle:* **YES** **NO**

Briefly explain. Your explanation must make reference to Tree 1 or Tree 2 explicitly, its root, and any relevant paths in the tree.

In Tree 1, there is at least one complete active path which tells us that the root of Tree 1, namely P, can be true $\therefore$ P is not a contradiction.

In tree 1, the complete active paths tell us

P is true when ① A is F, B is F

when ② C is F

when ③ B is T and C is T.

3c. Based on one of your two trees, give a disjunctive normal form for Y.

DNF for P**:**

$(\neg A \wedge \neg B) \vee (\neg C) \vee (B \wedge C)$

Which tree did you use to find your DNF for P? *Circle:* **Tree 1** **Tree 2**