

Assignment 1: ELECTRICITY

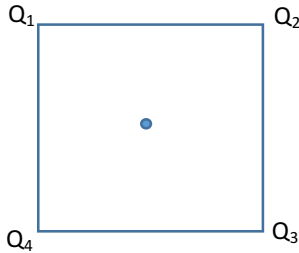
STUDENT #: _____

Released: Friday Jan. 12

Due: Fri Friday Jan 19 6:00PM

NAME: _____

- 1 4 charges of $Q_1=2\text{nC}$, $Q_2=4\text{nC}$, $Q_3=6\text{nC}$, and $Q_4=8\text{nC}$ are placed at the vertices of a square of side 1m. Q_1 is in the top left corner, Q_2 is in the top right corner, Q_3 is in the bottom right corner and the Q_4 in the bottom left corner. Find the magnitude of electric field at the center of the square. On the $Q_4 - Q_2$ diagonal the Field E is pointing toward Q_2 and has magnitude of 36N. On the $Q_1 - Q_3$ diagonal the Field E is pointing toward Q_1 and has magnitude of 36N.



The field direction is straight up (parallel to the Q_1-Q_3)

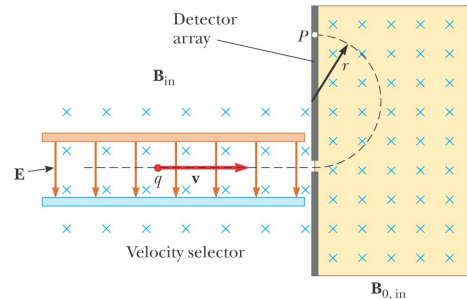
The magnitude of the force is given by:

$$\sqrt{(36\cos 45)^2 + 36(\cos 45)^2} = 36\text{N}$$

- 2A X-Ray lamp accelerates the electrons in the potential difference of 3kV. Find the lowest wavelength of the X rays emitted by this lamp.

$$eV = h \frac{c}{\lambda_{\min}} \Rightarrow \lambda_{\min} = h \frac{c}{eV} = 6.626 \times 10^{-34} \frac{2.998 \times 10^8}{3000 \times 1.6 \times 10^{-19}} \text{m} = 0.0041 \times 10^{-7} \text{m} = 0.41 \text{nm}$$

- 2B In the standard Mass Spectrometer Setup before entering the magnetic field zone where its trajectory is determined by the Lorentz Force the charged particles pass through the velocity selector. In it, the electric force acts on particles in the direction opposite to the magnetic force. After travelling along the straight line in the velocity selector employing E-field of magnitude of 2000N/C, the carbon ion ($^{12}\text{C}^+$) travels inside the spectrometer along the circular path of radius $r = 0.5\text{m}$. Find the velocity of the ion as it leaves the velocity selector.



SOLUTION:

In the Wien's Filter:

$$eE = evB$$

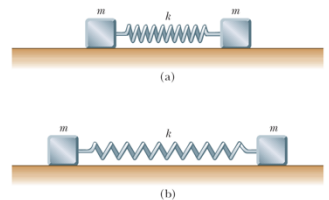
In the Magnetic field zone:

$$evB = \frac{mv^2}{r}$$

Combining the two equations: $evB = \frac{mv^2}{r} \Rightarrow eB = \frac{mv}{r} \Rightarrow \frac{eE}{v} = \frac{mv}{r} \Rightarrow mv^2 = eEr$

$$\Rightarrow v = \sqrt{\frac{eEr}{m}} = \sqrt{0.12 \times 10^{11}} \text{m/s} = 109.5 \text{ km/s}$$

- 3 Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having the spring constant 100 N/m and an unstretched length of 0.300 m as shown in Figure P19.57a. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length of 0.400 m as shown. Determine the value of Q, assuming that all the charge resides on the blocks and modeling the blocks as point charges.



Charge $\frac{Q}{2}$ resides on each block, which repel as point charges:

$$F = \frac{k_e (Q/2)(Q/2)}{L^2} = k(L - L_i). \text{ Solving for } Q,$$

$$Q = 2L \sqrt{\frac{k(L - L_i)}{k_e}}$$

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CONT.

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- 4 Two small spheres of mass m are suspended from strings of length ℓ that are connected at a common point. One sphere has charge Q , and the other has charge $2Q$. The strings make angles θ_1 and θ_2 with the vertical.

(a) How are θ_1 and θ_2 related?

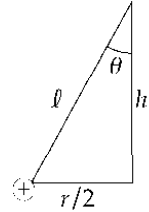
(b) Assume that θ_1 and θ_2 are small. Show that the distance r between the spheres is given by $r \approx \left(\frac{4k_e Q^2 \ell}{mg} \right)^{1/3}$

(a) From the $2Q$ charge we have $F_e - T_2 \sin \theta_2 = 0$ and $mg - T_2 \cos \theta_2 = 0$.

Combining these we find $\frac{F_e}{mg} = \frac{T_2 \sin \theta_2}{T_2 \cos \theta_2} = \tan \theta_2$.

From the Q charge we have $F_e = T_1 \sin \theta_1 = 0$ and $mg - T_1 \cos \theta_1 = 0$.

Combining these we find $\frac{F_e}{mg} = \frac{T_1 \sin \theta_1}{T_1 \cos \theta_1} = \tan \theta_1$ or $\boxed{\theta_2 = \theta_1}$.



(b) $F_e = \frac{k_e 2Q Q}{r^2} = \frac{2k_e Q^2}{r^2}$

If we assume θ is small then $\tan \theta = \sin \theta = \frac{r}{l}$. Substitute expressions for F_e and $\tan \theta$ into either equation found in part (a) and solve for r .

$$\frac{F_e}{mg} = \tan \theta \text{ then } \frac{2k_e Q^2}{r^2} \left(\frac{1}{mg} \right) \approx \frac{r}{l} \text{ and solving for } r \text{ we find } r = \sqrt[3]{\frac{4k_e \ell Q^2}{mg}}$$

- 5 Find the electric field at the point 20 cm above the center of the square made of 4 charged rods of $L=20\text{cm}$ and $Q=1\text{mC}$ each. (use the opposite page to present your solution with full diagram) Based on geometry the field at this point is equal to 4 times the normal components of the $|E|$ from single rod at distance $\sqrt{500}\text{cm}$.

$$E = 4 \cos \theta E_1 = 4 \frac{20}{\sqrt{500}} E_1 = \frac{8}{\sqrt{5}} 2k_e \lambda l \left(\frac{1}{y \sqrt{l^2 + y^2}} \right) = \frac{16 \cdot 8.99 \cdot 10^9 \cdot 10^{-3}}{\sqrt{5} \cdot 0.2 \sqrt{0.04 + 0.04}} = 1.14 \cdot 10^9 \text{ N/C}$$

- 6 Given is the rod of length L with the linear charge of density $\lambda = \frac{Q}{L}$. The rod lies on the x axis with its midpoint at the origin. Find the electric field vector on y axis resulting from such continuous system of charge at distance y from the origin.

Use this result to obtain the expression for electric field at distance y from the infinitely long wire.

Use opposite side of this page for your solution. Make sure that you include the large diagram with clearly stated variables, angles etc.

CONSULT YOUR NOTES FOR THE SOLUTION (NOTE TO TAs this is a Standard Problem I solved in class – 90% of students should have the correct answers)